1.1 $\vec{u} = \vec{a}_1 + 2\vec{a}_2 + 3\vec{a}_3$

$\vec{v} = \vec{b}_1 + \vec{c}_2 + \vec{d}_3$

$\vec{w} = \vec{d}_1 + 2\vec{d}_2 + q_3\vec{d}_3$

A	$\vec{u}$	$\vec{v}$	$\vec{w}$
q_1	N	Y	Y
q_2	N	Y	Y
q_3	N	~~Y~~ N	Y

B	$\vec{u}$	$\vec{v}$	$\vec{w}$
q_1	Y	N	N
q_2	N	Y	~~N~~ Y
q_3	N	N	Y

C	$\vec{u}$	$\vec{v}$	$\vec{w}$
q_1	Y	N	N
q_2	Y	Y	N
q_3	N	N	Y

D	$\vec{u}$	$\vec{v}$	$\vec{w}$
q_1	Y	N	N
q_2	Y	Y	N
q_3	Y	Y	Y

1.2 $\vec{V}$ function of t = $\vec{V}$ function of $\vec{q}_1$

These is the case at reference A.

1.3 $\vec{V}$ func of $t \Rightarrow \vec{V}$ func of $\vec{q}_2$

This is the case at reference A B C D

1.4 $\vec{v} = v_1\vec{a}_1 + v_2\vec{a}_2 + v_3\vec{a}_3$

$$v_1 = \frac{[\vec{v}\ \vec{a}_2\ \vec{a}_3]}{[\vec{a}_1\ \vec{a}_2\ \vec{a}_3]} = \frac{\vec{v}\cdot\vec{a}_2\times\vec{a}_3}{\vec{a}_1\cdot\vec{a}_2\times\vec{a}_3}$$

since $\vec{a}_1$ and $\vec{a}_2$ and $\vec{a}_3$ are non coplanar

$\vec{a}_2\times\vec{a}_3 = \vec{a}_1$ which gives us

$$v_1 = \frac{\vec{v}\cdot\vec{a}_1}{\vec{a}_1\cdot\vec{a}_1}$$ which is by definition v_1.

1.5 $\left|\frac{{}^A\partial\vec{V}}{\partial q_1}\right|$

$\left|\frac{{}^B\partial\vec{V}}{\partial q_1}\right| = 0$

$\frac{{}^C\partial\vec{V}}{\partial q_2}$; ${}^C\vec{V} = \overbrace{cq_2\vec{c}_1 + sq_2\vec{c}_3}^{\vec{b}_1} + \vec{c}_2 + \overbrace{\vec{c}_3}^{\vec{d}_3}$

$\frac{{}^C\partial\vec{V}}{\partial q_2} = -sq_2\vec{c}_1 + cq_2\vec{c}_3$

$\left|\frac{{}^C\partial\vec{V}}{\partial q_2}\right| = sq_2^2 + cq_2^2 = 1$

$\left|\frac{{}^C\partial\vec{V}}{\partial q_3}\right| = 0$ Notice that ${}^C\vec{V}$ is not dependent on q_3

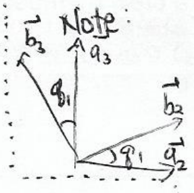
Note: ${}^D\vec{V} =$

$\left|\frac{{}^D\partial\vec{V}}{\partial q_2}\right|$

$\left|\frac{{}^D\partial\vec{V}}{\partial q_1}\right| = 0$ ${}^D\vec{V}$ is not dependent on $\vec{q}_1$

1.6 ${}^B\vec{u} = \vec{a}_1 + 2\vec{a}_2 + 3\vec{a}_3$

${}^B\vec{u} = \vec{b}_1 + 2cq_1\vec{b}_2 - 2sq_1\vec{b}_3 + 3sq_1\vec{b}_2 + 3cq_1\vec{b}_3$



Note:

$\vec{a}_1 = \vec{b}_1$

$\vec{b}_2 = cq_1\vec{a}_2 + sq_1\vec{a}_3$

$\vec{b}_3 = cq_1\vec{a}_3 - sq_1\vec{a}_2$

$\vec{a}_2 = cq_1\vec{b}_2 - sq_1\vec{b}_3$

$\vec{a}_3 = cq_1\vec{b}_3 + sq_1\vec{b}_2$

$\frac{\partial^B\vec{u}}{\partial q_1} = -2sq_1\vec{b}_2 - 2cq_1\vec{b}_3 + 3cq_1\vec{b}_2 - 3sq_1\vec{b}_3$

convert $\vec{b}_2$ and $\vec{b}_3$ to $\vec{a}_1, \vec{a}_2, \vec{a}_3$

$= \cancel{-2sq_1cq_1\vec{a}_2} - 2sq_1sq_1\vec{a}_3$
$-2cq_1cq_1\vec{a}_3 + \cancel{2cq_1sq_1\vec{a}_2}$
$\cancel{-3sq_1cq_1\vec{a}_3} + 3sq_1sq_1\vec{a}_2$
$+3cq_1cq_1\vec{a}_2 + \cancel{3cq_1sq_1\vec{a}_3}$

$= -2\vec{a}_3 + 3\vec{a}_2$

(a) 0, 3, −2

(b) $0,\ -2sq_1 + 3cq_1,\ -2cq_1 - 3sq_1$

(c) 0, 0, 0 bec ${}^A\vec{u}$ is not dependent on q_1

1.7

1.9 Note:

$$\vec{a}_1 = c\theta\,\vec{b}_1 - s\theta\,\vec{b}_2$$

$$\vec{a}_2 = s\theta\,\vec{b}_1 + c\theta\,\vec{b}_2$$

$${}^A\vec{V} = f\vec{a}_1 + g\vec{a}_2$$

$${}^B\vec{V} = f\left(c\theta\vec{b}_1 - s\theta\vec{b}_2\right) + g\left(s\theta\vec{b}_1 + c\theta\vec{b}_2\right)$$

$$\frac{{}^B\partial\vec{V}}{\partial q_2} = \frac{\partial f}{\partial q_2}\left(c\theta\vec{b}_1 - s\theta\vec{b}_2\right) + f\left(-s\theta\vec{b}_1 - c\theta\vec{b}_2\right)\frac{\partial\theta}{\partial q_2} + \frac{\partial g}{\partial q_2}\left(s\theta\vec{b}_1 + c\theta\vec{b}_2\right) + g\left(c\theta\vec{b}_1 - s\theta\vec{b}_2\right)\frac{\partial\theta}{\partial q_2}$$

$$\frac{{}^A\partial}{\partial q_1}\left(\frac{{}^B\partial\vec{V}}{\partial q_2}\right) = \frac{{}^A\partial}{\partial q_1}\left(\frac{\partial f}{\partial q_2}\vec{a}_1 - f\vec{a}_2\frac{\partial\theta}{\partial q_2} + \frac{\partial g}{\partial q_2}\vec{a}_2 + g\vec{a}_1\frac{\partial\theta}{\partial q_2}\right)$$

$$= \frac{\partial^2 f}{\partial q_1\partial q_2}\vec{a}_1 - \frac{\partial f}{\partial q_1}\frac{\partial\theta}{\partial q_2}\vec{a}_2 - f\frac{\partial^2\theta}{\partial q_1\partial q_2}\vec{a}_2 + \frac{\partial^2 g}{\partial q_1\partial q_2}\vec{a}_2 + \frac{\partial g}{\partial q_1}\frac{\partial\theta}{\partial q_2}\vec{a}_1 + g\frac{\partial^2\theta}{\partial q_1\partial q_2}\vec{a}_1$$

$$\frac{{}^A\partial\vec{V}}{\partial q_1} = \frac{\partial f}{\partial q_1}\vec{a}_1 + \frac{\partial g}{\partial q_1}\vec{a}_2$$

$$\frac{{}^B\partial}{\partial q_2}\left(\frac{{}^A\partial\vec{V}}{\partial q_1}\right) = \frac{\partial^2 f}{\partial q_1\partial q_2}\vec{a}_1 + \frac{\partial f}{\partial q_1}\frac{\partial\theta}{\partial q_2}\left(-\vec{a}_2\right) + \frac{\partial^2 g}{\partial q_1\partial q_2}\vec{a}_2 + \frac{\partial g}{\partial q_1}\frac{\partial\theta}{\partial q_2}\left(\vec{a}_1\right)$$

$$\frac{{}^A\partial}{\partial q_1}\left(\frac{{}^B\partial\vec{V}}{\partial q_2}\right) - \frac{{}^B\partial}{\partial q_2}\left(\frac{{}^A\partial\vec{V}}{\partial q_1}\right) = -f\frac{\partial^2\theta}{\partial q_1\partial q_2}\vec{a}_2 + g\frac{\partial^2\theta}{\partial q_1\partial q_2}\vec{a}_1$$

$$= \left(g\vec{a}_1 - f\vec{a}_2\right)\frac{\partial^2\theta}{\partial q_1\partial q_2} \quad \blacksquare$$

1.8 (a)

$$\vec{V}\cdot\frac{d\vec{V}}{dt} = \sum v_i\vec{a}_i\cdot\sum\frac{dv_i}{dt}\vec{a}_i$$

$$= \sum v_i\frac{dv_i}{dt}$$

(b)

$$|\vec{V}|\,\frac{d|\vec{V}|}{dt} = \left(\sqrt{\sum v_i^2}\right)\left(\tfrac{1}{2}\left(\sum v_i^2\right)^{-1/2}\left(2\sum v_i\frac{dv_i}{dt}\right)\right)$$

$$= \sum v_i\frac{dv_i}{dt}$$

since (a) == (b), we just showed

$$|\vec{V}|\,\frac{d|\vec{V}|}{dt} = \vec{V}\cdot\frac{d\vec{V}}{dt}$$

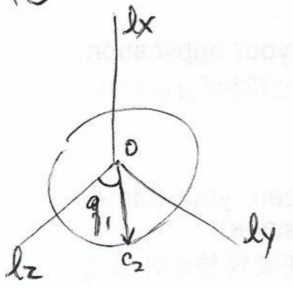
$$\vec{u} = \frac{\vec{V}}{|V|}$$

$$\rightarrow -\tfrac{1}{2}\left(\sum v_i^2\right)^{-3/2}\left(\sum 2v_i\frac{dv_i}{dt}\right) = -|V|^{-3}\vec{v}\cdot\frac{d\vec{v}}{dt}$$

$$\frac{d\vec{u}}{dt} = \frac{d\vec{V}}{dt}|V|^{-1} + \frac{d|V|^{-1}}{dt}\vec{V}$$

$$\boxed{= \frac{d\vec{V}}{dt}|V|^{-1} - |V|^{-3}\vec{v}\cdot\frac{d\vec{v}}{dt}\,\vec{V}}$$

1.10



$\vec{c}_1 = \vec{l}_x$

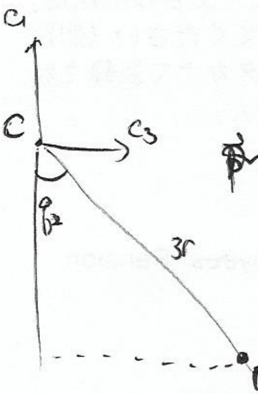
$\vec{c}_2 = -cq_1\vec{l}_z + sq_1\vec{l}_y$

$\vec{c}_3 = sq_1\vec{l}_z + cq_1\vec{l}_y$

$\vec{v}_{oc} = r\vec{c}_2$

$${}^L\frac{\partial \vec{c}_2}{\partial q_1} = sq_1\vec{l}_z + cq_1\vec{l}_y = \vec{c}_3$$

$${}^L\frac{\partial \vec{c}_3}{\partial q_1} = cq_1\vec{l}_z - sq_1\vec{l}_y = -\vec{c}_2$$



$\vec{v}_{cp} = 3r(-cq_2\vec{c}_1 + sq_2\vec{c}_3)$

$\vec{p} = \vec{v}_{oc} + \vec{v}_{cp}$

$\vec{p} = \vec{v}_{op} = -3rcq_2\vec{c}_1 + r\vec{c}_2 + 3rsq_2\vec{c}_3$

Note:

$${}^L\frac{\partial \vec{c}_1}{\partial t} = 0 \qquad {}^L\frac{\partial \vec{c}_2}{\partial t} = {}^L\frac{\partial \vec{c}_2}{\partial q_1}\dot{q}_1 = \vec{c}_3\dot{q}_1$$

$${}^L\frac{\partial \vec{c}_3}{\partial t} = {}^L\frac{\partial \vec{c}_3}{\partial q_1}\dot{q}_1 = -\vec{c}_2\dot{q}_1$$

$${}^L\frac{\partial \vec{p}}{\partial t} = r\left[3sq_2\dot{q}_2\vec{c}_1 - \cancel{0} + 3cq_2\dot{q}_2\vec{c}_3 - 3sq_2\dot{q}_1\vec{c}_2 + \dot{q}_1\vec{c}_3\right]$$

$$= r\left[\dot{q}_1(\vec{c}_3 - 3sq_2\vec{c}_2) + 3\dot{q}_2(sq_2\vec{c}_1 + cq_2\vec{c}_3)\right]$$

■

1.11 $\vec{u} = \frac{\vec{v}}{|v|}$

$$\frac{d\vec{u}}{dt} = \frac{d\vec{v}}{dt}|v|^{-1} + \vec{v}\frac{d|v|^{-1}}{dt}$$

$$\frac{d^2\vec{u}}{dt^2} = \frac{d^2\vec{v}}{dt^2}|v|^{-1} + 2\frac{d\vec{v}}{dt}\frac{d|v|^{-1}}{dt} + \vec{v}\frac{d^2|v|^{-1}}{dt^2}$$

Note:

$$\frac{d|v|^{-1}}{dt} = -\tfrac{1}{2}\left(\sum v_i^2\right)^{-3/2}\sum 2v_i\frac{dv_i}{dt} = -|v|^{-3}\underbrace{\vec{v}\cdot\frac{d\vec{v}}{dt}}_{0} = 0$$

$$\frac{d^2|v|^{-1}}{dt^2} = \frac{d\left(-|v|^{-3}\right)}{dt}\vec{v}\cdot\frac{d\vec{v}}{dt} + \left(-|v|^{-3}\right)\frac{d\left(\vec{v}\cdot\frac{d\vec{v}}{dt}\right)}{dt}$$

$$= -\left(-\tfrac{3}{2}\right)\left(\sum v_i^2\right)^{-5/2}\sum 2v_i\frac{dv_i}{dt}\sum v_i\frac{dv_j}{dt} - |v|^{-3}\left(\sum\frac{dv_i}{dt}\frac{dv_i}{dt} + \sum v_i\frac{d^2v_i}{dt^2}\right)$$

$$= -3|v|^{-5}\Big(\underbrace{\vec{v}\cdot\frac{d\vec{v}}{dt}}_{0}\Big)^2 - \underbrace{|v|^{-3}}_{1}\Big(\underbrace{\frac{d\vec{v}}{dt}\cdot\frac{d\vec{v}}{dt}}_{1} + \underbrace{\vec{v}\cdot\frac{d^2\vec{v}}{dt^2}}_{0}\Big)$$

$$\frac{d^2\vec{u}}{dt^2} = \vec{n}_3 * 1 + 2\vec{n}_2 * 0 + \vec{n}_1 * 1 = \vec{n}_3 + \vec{n}_1$$

$$\boxed{\left|\frac{d^2\vec{u}}{dt^2}\right| = \sqrt{2}}$$ //