2.1 By definition ${}^A\vec{\omega}^B = \vec{b}_1 \frac{{}^A d\vec{b}_2}{dt}\cdot\vec{b}_3 + \vec{b}_2 \frac{{}^A d\vec{b}_3}{dt}\cdot\vec{b}_1 + \vec{b}_3 \frac{{}^A d\vec{b}_1}{dt}\cdot\vec{b}_2$

To express ${}^A\vec{\omega}^B$ in terms of $\vec{b}_1\ \vec{b}_2\ \vec{b}_3\ \rho\ \lambda$ and $\dot{s}$, I must evaluate $\frac{{}^A d\vec{b}_i}{dt}$ for $i = 1$ to 3

Note (given in the problem) $\frac{{}^A d\vec{b}_i}{dt} = \vec{b}_i{}'\dot{s}$ for $i = 1, 2, 3$

$${}^A\vec{\omega}^B = \vec{b}_1\left(-\frac{\vec{b}_1}{\rho} + \lambda\vec{b}_3\right)\dot{s}\cdot\vec{b}_3 + \vec{b}_2\left(-\lambda\vec{b}_2\right)\dot{s}\cdot\vec{b}_1 + \vec{b}_3\left(\frac{\vec{b}_2}{\rho}\dot{s}\right)\cdot\vec{b}_2$$

Note that since $\vec{b}_1, \vec{b}_2, \vec{b}_3$ are orthogonal, dot product ~~between~~ $\vec{b}_i\cdot\vec{b}_j$ for $i \neq j$ equals 0

$$\boxed{{}^A\vec{\omega}^B = \vec{b}_1(\lambda\dot{s}) + \vec{b}_3\frac{\dot{s}}{\rho}}$$

2.2 ${}^A\vec{\omega}^B = \vec{b}_1\frac{{}^A d\vec{b}_2}{dt}\cdot\vec{b}_3 + \vec{b}_2\frac{{}^A d\vec{b}_3}{dt}\cdot\vec{b}_1 + \vec{b}_3\frac{{}^A d\vec{b}_1}{dt}\cdot\vec{b}_2$

α_2 = take the $\vec{a}_2$ components of $\vec{b}_i$ for $i = 1,2,3$

$$= \left[\frac{{}^A d\vec{b}_2}{dt}\cdot\vec{b}_3\,(c_2 s_3) + \frac{{}^A d\vec{b}_3}{dt}\cdot\vec{b}_1\,(s_1 s_2 s_3 + c_3 c_1) + \frac{{}^A d\vec{b}_1}{dt}\cdot\vec{b}_2\,(c_1 s_2 s_3 - c_3 s_1)\right]$$

Note:

$$\frac{{}^A d\vec{b}_1}{dt} = (-s_2 c_3\dot{q}_2 - c_2 s_3\dot{q}_3)\,\vec{a}_1 + (-s_2 s_3\dot{q}_2 + c_2 c_3\dot{q}_3)\,\vec{a}_2 + (-c_2\dot{q}_2)\,\vec{a}_3$$

$$\frac{{}^A d\vec{b}_2}{dt} = (c_1 s_2 c_3\dot{q}_1 + s_1 c_2 c_3\dot{q}_2 - s_1 s_2 s_3\dot{q}_3 - c_3 c_1\dot{q}_3 + s_3 s_1\dot{q}_1)\,\vec{a}_1 + (c_1 s_2 s_3\dot{q}_1 + s_1 c_2 s_3\dot{q}_2 + s_1 s_2 c_3\dot{q}_3 - s_3 c_1\dot{q}_3 - c_3 s_1\dot{q}_1)\,\vec{a}_2 + (c_1 c_2\dot{q}_1 - s_1 s_2\dot{q}_2)\,\vec{a}_3$$

$$\frac{{}^A d\vec{b}_3}{dt} = (-s_1 s_2 c_3\dot{q}_1 + c_1 c_2 c_3\dot{q}_2 - c_1 s_2 s_3\dot{q}_3 + c_3 s_1\dot{q}_3 + s_3 c_1\dot{q}_1)\,\vec{a}_1 + (-s_1 s_2 s_3\dot{q}_1 + c_1 c_2 s_3\dot{q}_2 + c_1 s_2 c_3\dot{q}_3 + s_3 s_1\dot{q}_3 - c_3 c_1\dot{q}_1)\,\vec{a}_2 + (-s_1 c_2\dot{q}_1 - c_1 s_2\dot{q}_2)\,\vec{a}_3$$

$$\beta_2 = \frac{{}^A d\vec{b}_3}{dt}\cdot\vec{b}_1 = -s_1 s_2 c_2\dot{q}_1 + c_1 c_2^2\dot{q}_2 + s_1 c_2\dot{q}_3 + s_1 c_2 s_2\dot{q}_1 + c_1 s_2^2\dot{q}_2 = \boxed{c_1\dot{q}_2 + s_1 c_2\dot{q}_3}\ \beta_2$$

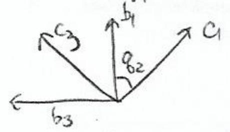
$$\beta_1 = c_1^2 s_2^2\dot{q}_1 + s_1 c_1 s_2 c_2\dot{q}_2 - c_1^2 s_2\dot{q}_3 - s_1^2 s_2\dot{q}_3 + s_1^2\dot{q}_1 + c_1^2 c_2^2\dot{q}_1 - s_1 c_1 s_2 c_2\dot{q}_2 = \dot{q}_1 - s_2\dot{q}_3$$

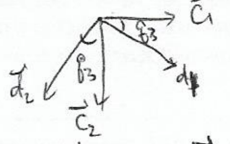
$$\beta_3 = -s_1 s_2^2\dot{q}_2 + c_1 c_2\dot{q}_3 - s_1 c_2^2\dot{q}_2 = -s_1\dot{q}_2 + c_1 c_2\dot{q}_3$$

$$\alpha_2 = c_2 s_3\dot{q}_1 - s_2 c_2 s_3\dot{q}_3 + s_1 c_1 s_2 s_3\dot{q}_2 + c_1^2 c_3\dot{q}_2 + s_1^2 s_2 c_2 s_3\dot{q}_3 + s_1 c_1 c_2 c_3\dot{q}_3 + -s_1 c_1 s_2 s_3\dot{q}_2 + s_1^2 c_3\dot{q}_2 + c_1^2 s_2 c_2 s_3\dot{q}_3 - s_1 c_1 c_2 c_3\dot{q}_3 = \boxed{c_2 s_3\dot{q}_1 + c_3\dot{q}_2}\ \alpha_2$$

2.3 $^{D}\vec{\omega}^{B} = \vec{b}_1 \frac{^{D}d\vec{b}_2}{dt}\cdot\vec{b}_3 + \vec{b}_2 \frac{^{D}d\vec{b}_3}{dt}\cdot\vec{b}_1 + \vec{b}_3 \frac{^{D}d\vec{b}_1}{dt}\cdot\vec{b}_2$

we need $\vec{b}_1, \vec{b}_2, \vec{b}_3$ in terms of $\vec{d}_1\ \vec{d}_2\ \vec{d}_3$





$\vec{b}_1 = cq_2\vec{c}_1 + sq_2\vec{c}_3$ $\quad \vec{c}_1 = cq_3\vec{d}_1 - sq_3\vec{d}_2$ $\quad \vec{b}_1 = cq_2cq_3\vec{d}_1 - cq_2sq_3\vec{d}_2 + sq_2\vec{d}_3$

$\vec{b}_2 = \vec{c}_2$ $\quad \vec{c}_2 = sq_3\vec{d}_1 + cq_3\vec{d}_2$ $\quad \vec{b}_2 = sq_3\vec{d}_1 + cq_3\vec{d}_2$

$\vec{b}_3 = -sq_2\vec{c}_1 + cq_2\vec{c}_3$ $\quad \vec{c}_3 = \vec{d}_3$ $\quad \vec{b}_3 = -sq_2cq_3\vec{d}_1 + sq_2sq_3\vec{d}_2 + cq_2\vec{d}_3$

$$\frac{^{D}d\vec{b}_1}{dt} = \left(-sq_2\dot{q}_2cq_3 - cq_2sq_3\dot{q}_3\right)\vec{d}_1 + \left(sq_2\dot{q}_2sq_3 - cq_2cq_3\dot{q}_3\right)\vec{d}_2 + cq_2\dot{q}_2\vec{d}_3$$

$$\frac{^{D}d\vec{b}_2}{dt} = cq_3\dot{q}_3\vec{d}_1 - sq_3\dot{q}_3\vec{d}_2$$

$$\frac{^{D}d\vec{b}_3}{dt} = \left(-cq_2cq_3\dot{q}_2 + sq_2sq_3\dot{q}_3\right)\vec{d}_1 + \left(cq_2sq_3\dot{q}_2 + sq_2cq_3\dot{q}_3\right)\vec{d}_2 - sq_2\dot{q}_2\vec{d}_3$$

$$^{D}\vec{\omega}^{B} = \vec{b}_1\left(-sq_2cq_3^2\dot{q}_3 - sq_2sq_3^2\dot{q}_3\right) + \vec{b}_2\left(-cq_2^2cq_3^2\dot{q}_2 + sq_2cq_2sq_3cq_3\dot{q}_3 - cq_2^2sq_3^2\dot{q}_2 - sq_2^2\dot{q}_2 - sq_2cq_2sq_3cq_3\dot{q}_3\right)$$
$$+ \vec{b}_3\left(-sq_2sq_3cq_3\dot{q}_2 - cq_2sq_3^2\dot{q}_3 + sq_2sq_3cq_3\dot{q}_2 - cq_2cq_3^2\dot{q}_3\right)$$

$$\boxed{^{D}\vec{\omega}^{B} = -sq_2\dot{q}_3\vec{b}_1 - \dot{q}_2\vec{b}_2 - cq_2\dot{q}_3\vec{b}_3}$$

- solving for $^{D}\vec{\omega}^{C} = \vec{c}_1\frac{^{D}d\vec{c}_2}{dt}\cdot\vec{c}_3 + \vec{c}_2\frac{^{D}d\vec{c}_3}{dt}\cdot\vec{c}_1 + \vec{c}_3\frac{^{D}d\vec{c}_1}{dt}\cdot\vec{c}_2$

$\frac{^{D}d\vec{c}_1}{dt} = -sq_3\dot{q}_3\vec{d}_1 - cq_3\dot{q}_3\vec{d}_2 \quad \frac{^{D}d\vec{c}_2}{dt} = cq_3\dot{q}_3\vec{d}_1 - sq_3\dot{q}_3\vec{d}_2 \quad \frac{^{D}d\vec{c}_3}{dt} = 0$

$^{D}\vec{\omega}^{C} = 0\,\vec{c}_1 + \vec{c}_2\,0 + \vec{c}_3\left(-sq_3^2\dot{q}_3 - cq_3^2\dot{q}_3\right) = \boxed{-\dot{q}_3\vec{c}_3 = {}^{D}\vec{\omega}^{C}}$

- Solving for $^{C}\vec{\omega}^{B} = \vec{b}_1\frac{^{C}d\vec{b}_2}{dt}\cdot\vec{b}_3 + \vec{b}_2\frac{^{C}d\vec{b}_3}{dt}\cdot\vec{b}_1 + \vec{b}_3\frac{^{C}d\vec{b}_1}{dt}\cdot\vec{b}_2$

$\frac{^{C}d\vec{b}_1}{dt} = -sq_2\dot{q}_2\vec{c}_1 + cq_2\dot{q}_2\vec{c}_3 \quad \frac{^{C}d\vec{b}_2}{dt} = 0 \quad \frac{^{C}d\vec{b}_3}{dt} = -cq_2\dot{q}_2\vec{c}_1 - sq_2\dot{q}_2\vec{c}_3$

$^{C}\vec{\omega}^{B} = 0\,\vec{b}_1 + -\dot{q}_2\vec{b}_2 + 0\,\vec{b}_3 \Rightarrow \boxed{^{C}\vec{\omega}^{B} = -\dot{q}_2\vec{b}_2}$

- show that $^{D}\vec{\omega}^{B} = {}^{D}\vec{\omega}^{C} + {}^{C}\vec{\omega}^{B}$ $\quad$ Note: $\vec{c}_3 = sq_2\vec{b}_1 + cq_2\vec{b}_3$

$= -\dot{q}_3\vec{c}_3 - \dot{q}_2\vec{b}_2 = ~~-\dot{q}_3\vec{d}_3 - sq_3\dot{q}_2\vec{d}_1 - cq_3\dot{q}_2\vec{d}_2~~$

$= -\dot{q}_3sq_2\vec{b}_1 - \dot{q}_3cq_2\vec{b}_3 - \dot{q}_2\vec{b}_2$ which is equal to $^{D}\vec{\omega}^{B}$ ∎

- Solve for $\omega_i \triangleq {}^{A}\vec{\omega}^{D}\cdot\vec{d}_i$

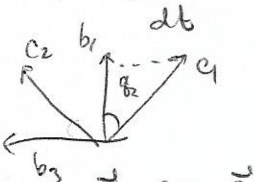
$^{A}\vec{\omega}^{D} = {}^{A}\vec{\omega}^{B} + {}^{B}\vec{\omega}^{C} + {}^{C}\vec{\omega}^{D} = \dot{q}_1\vec{b}_1 + \dot{q}_2\vec{c}_2 + \dot{q}_3\vec{d}_3$

$= cq_2cq_3\dot{q}_1\vec{d}_1 - cq_2sq_3\dot{q}_1\vec{d}_2 + sq_2\dot{q}_1\vec{d}_3 + sq_3\dot{q}_2\vec{d}_1 + cq_3\dot{q}_2\vec{d}_2 + \dot{q}_3\vec{d}_3$

$$\boxed{\begin{aligned} \omega_1 &= cq_2cq_3\dot{q}_1 + sq_3\dot{q}_2 \\ \omega_2 &= -cq_2sq_3\dot{q}_1 + cq_3\dot{q}_2 \\ \omega_3 &= \dot{q}_3 + sq_2\dot{q}_1 \end{aligned}}$$

//

$^{D}\omega^{B}$ $D \frac{d\vec{b}_{1,2,3}}{dt}$ $\vec{b}_1 =$ $\vec{d}_1$ $\vec{d}_2$ $\vec{d}_3$



$\vec{a}_1 = a_1 b_1 \vec{b}_1$ $\vec{b}_2$ $\vec{b}_3$

$^{A}\omega^{B} = \vec{b}_1 \frac{^{A}d\vec{b}_2}{dt} \vec{b}_3 + \vec{b}_2 \frac{d\vec{b}_3}{dt} \vec{b}_1 + \vec{b}_3 \frac{d\vec{b}_1}{dt} \vec{b}_2$

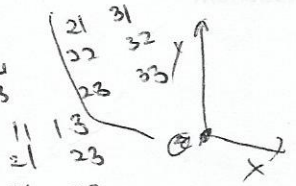
$^{B}\omega^{A} = \vec{a}_1 \frac{^{B}d\vec{a}_2}{dt} \vec{a}_3 + \vec{a}_2 \frac{d\vec{a}_3}{dt} \vec{a}_1 + \vec{a}_3 \frac{d\vec{a}_1}{dt} \vec{a}_2$

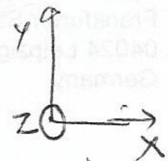
$\vec{b}_1 = cq_2 \vec{c}_1 + sq_2 \vec{c}_2$

$\vec{b}_3 = -sq_2 \vec{c}_1 + cq_2 \vec{c}_2$

$\vec{b}_2 = \vec{c}_2$

$\vec{c}_3 = \vec{d}_3$





$^{A}\omega^{B} = \dot{q}_1 \vec{b}_1$

$^{B}\omega^{C} = \dot{q}_2 \vec{c}_2$

$^{C}\omega^{D} = \dot{q}_3 \vec{d}_3$

$^{A}\omega^{D} = \dot{q}_1 \vec{b}_1 + \dot{q}_2 \vec{c}_2 + \dot{q}_3 \vec{d}_3$

$\dot{q}_1 \vec{b}_1$

$r_{11}\left(\sum r_{2i}' * r_{31}\right) + r_{21}\left(\sum r_{3i}' * r_{1i}\right) + r_{31}\left(\sum r_{1i}' * r_{21}\right)$

$\vec{b}_1\left(\sum r_{i1}' * r_{i3}\right) + \vec{b}_2\left(\sum r_{i3}' * r_{i1}\right) + \vec{b}_3\left(\sum r_{i1}' * r_{i2}\right)$

$cq_2 cq_3 \dot{q}_1 \vec{d}_1 - cq_2 sq_3 \dot{q}_1 \vec{d}_2 + sq_2 \dot{q}_1 \vec{d}_3 + sq_3 \dot{q}_2 \vec{d}_1 + cq_3 \dot{q}_2 \vec{d}_2 + \dot{q}_3 \vec{d}_3$

$\vec{a}_1 = r_{11} \vec{b}_1 + r_{12} \vec{b}_2 + r_{13} \vec{b}_3$ $r_{ij}' = \frac{dr_{ij}}{dt}$ $\vec{b}_1 = r_{11} \vec{a}_1 + r_{21} \vec{a}_2 + r_{31} \vec{a}_3$

$\vec{a}_1\left(\sum r_{2i}' * r_{31}\right) + \vec{a}_2\left(\sum r_{3i}' * r_{1i}\right) + \vec{a}_3\left(\sum r_{1i}' * r_{2i}\right)$

$\frac{^{A}d\vec{b}_3}{dt} = -s_1 s_2 c_3 \dot{q}_1 + c_1 c_2 c_3 \dot{q}_2 - c_1 s_2 s_3 \dot{q}_3 + c_3 s_1 \dot{q}_3 + s_3 c_1 \dot{q}_1 \; \vec{a}_1$ $c_2 c_3$

$-s_1 s_2 s_3 \dot{q}_1 + c_1 c_2 s_3 \dot{q}_2 + c_1 s_2 c_3 \dot{q}_3 + c_3 s_1 \dot{q}_3 - c_3 c_1 \dot{q}_1 \; \vec{a}_2$ $c_2 s_3$

$-s_1 c_2 \dot{q}_1 + c_1 s_2 \dot{q}_2 \; \vec{a}_3$

$\beta_2 = \dot{q}_2 \frac{^{A}d\vec{b}_3}{dt} \cdot \vec{b}_1$

$^{A}\omega^{B} =$

$X_L = R X_G$

$X_G = R^T X_L$

$X_L = r_{11} X_G + r_{12} Y_G + r_{13} Z_G$

Y_L

Z_L

$X_G = r_{11} X_L + r_{21} Y_L + r_{31} Z_L$

$^{A}\omega^{B} \times {}^{A}\omega^{B} \times \vec{p}$ (= 0)

$\vec{p}' = \frac{d\vec{p}}{dt} = {}^{A}\omega^{B} \times \vec{p}$

$\vec{p}'' = \frac{d\vec{p}'}{dt} = {}^{A}\omega^{B} \times \vec{p}'$

$^{A}\omega^{B} \times \vec{b}_1$

a b c

a b c

$\frac{^{A}d\vec{p}}{dt} = {}^{A}\omega^{B} \times \vec{p}$

$^{A}\omega^{B} = \vec{b}_1 \frac{^{A}d\vec{b}_2}{dt} \cdot \vec{b}_3 + \vec{b}_2 \frac{^{A}d\vec{b}_3}{dt} \vec{b}_1$

$\frac{^{A}d\vec{b}_2}{dt} = {}^{A}\omega^{B} \times \vec{b}_2$

$\frac{^{A}d\vec{b}_1}{dt}\cdot\vec{b}_2$	$\frac{^{A}d\vec{b}_3}{dt}\vec{b}_1$	$\frac{^{A}d\vec{b}_2}{dt}\vec{b}_3$
0	$-\vec{b}_2 +$	0
$\frac{^{A}d\vec{b}_2}{dt}\vec{b}_1$		$\frac{^{A}d\vec{b}_3}{dt}\vec{b}_2$

$(c_1 s_2 c_3 + s_3 s_1)(c_1 s_2 c_3 + s_3 s_1)\dot{q}_1 + (s_1 c_2 c_3)(c_1 s_2 c_3 + s_3 s_1)\dot{q}_2$

$-(s_1 s_2 s_3 + c_3 c_1)(c_1 s_2 c_3 + s_3 s_1)\dot{q}_3$

$+(c_1 s_2 s_3 - c_3 s_1)(c_1 s_2 s_3 - c_3 s_1)\dot{q}_1 + (s_1 c_2 s_3)(c_1 s_2 s_3 - c_3 s_1)\dot{q}_2$

$+(s_1 s_2 c_3 - s_3 c_1)(c_1 s_2 s_3 - c_3 s_1)\dot{q}_3$

$c_1 c_2 \dot{q}_1 c_1 c_2 - s_2 \dot{q}_2 c_1 c_2$

2.5 Note:

Approach 1 : $\frac{d(\vec{\beta}_1\cdot\vec{\beta}_2)}{dt}=0$ since $\vec{\beta}_1$ and $\vec{\beta}_2$ are fixed in a rigid body B

① $\dot{\vec{\beta}}_1\cdot\vec{\beta}_2+\vec{\beta}_1\dot{\vec{\beta}}_2=0 \Rightarrow \dot{\vec{\beta}}_1\vec{\beta}_2=-\vec{\beta}_1\dot{\vec{\beta}}_2$

Approach 2 $\vec{\beta}_2\cdot(\vec{\omega}\times\vec{\beta}_1)=\vec{\beta}_1\cdot(\vec{\beta}_2\times\vec{\omega})=\omega\cdot(\vec{\beta}_1\times\vec{\beta}_2)$ prop. of $\times$ and .

① $\vec{\beta}_2\cdot\dot{\vec{\beta}}_1=\vec{\beta}_1\cdot(-\dot{\vec{\beta}}_2)=-\vec{\beta}_1\dot{\vec{\beta}}_2$

golden rule $\frac{^{A}d\vec{\beta}}{dt}=\vec{\omega}\times\vec{\beta}$

$\vec{\beta}_2\times\vec{\omega}=-\vec{\omega}\times\vec{\beta}_2$

Given eq ① (assume all variables below are vectors)

$$\frac{1}{2}\left(\frac{\dot{\beta}_1\times\dot{\beta}_2}{\dot{\beta}_1\cdot\beta_2}+\frac{\dot{\beta}_2\times\dot{\beta}_1}{\dot{\beta}_2\cdot\beta_1}\right)=\frac{1}{2}\left(\frac{\dot{\beta}_1\times\dot{\beta}_2}{\dot{\beta}_1\beta_2}+\frac{-\dot{\beta}_1\times\dot{\beta}_2}{-\dot{\beta}_1\beta_2}\right)$$

$$=\frac{\dot{\beta}_1\times\dot{\beta}_2}{\dot{\beta}_1\beta_2}$$ ∎

2.6 attempt #1

$$\frac{d\vec{r}}{dt} = \frac{d\vec{p}}{dt} + \frac{d\vec{q}}{dt}$$

$$= \vec{b}_1 + \dot{q}_1\vec{b}_1 + \dot{q}_2\vec{b}_2 + \dot{q}_3\vec{b}_3$$ ✗ wrong because the derivative is in reference frame A

attempt # 2

$$\frac{{}^A d\vec{r}}{dt} = \frac{{}^A d\vec{p}}{dt} + \frac{{}^A d\vec{q}}{dt}$$

Note ① that $\vec{b}_1 = \frac{{}^A d\vec{p}}{ds}$ multiply $\dot{s}$ on both sides gives us $\dot{s}\,\vec{b}_1 = \frac{d\vec{p}}{dt}$

Note ② $\frac{{}^A d\vec{q}}{dt} = \frac{{}^B d\vec{q}}{dt} + {}^A\vec{\omega}^B \times \vec{q}$ where ${}^A\omega^B = \lambda\dot{s}\,\vec{b}_1 + \frac{\dot{s}}{\rho}\vec{b}_3$ (from solving P 2.1)

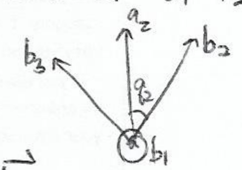
$$= \dot{q}_1\vec{b}_1 + \dot{q}_2\vec{b}_2 + \dot{q}_3\vec{b}_3 + \left(-\frac{\dot{s}}{\rho}q_2\right)\vec{b}_1 + \left(\frac{\dot{s}}{\rho}q_1 - \lambda\dot{s}q_3\right)\vec{b}_2 + \left(\lambda\dot{s}q_2\right)\vec{b}_3$$

$$\frac{{}^A d\vec{r}}{dt} = \left(\dot{q}_1 + \dot{s}\left(1 - \frac{q_2}{\rho}\right)\right)\vec{b}_1 + \left(\dot{q}_2 + \dot{s}\left(\frac{q_1}{\rho} - \lambda q_3\right)\right)\vec{b}_2 + \left(\dot{q}_3 + \lambda\dot{s}q_2\right)\vec{b}_3$$

2.7

$${}^{A}\vec{\omega}^{C} = \dot{q}_1 \vec{a}_z - \dot{q}_2 \vec{b}_1 + \dot{q}_3 \vec{b}_3$$ (obtain by observation/ aligning frame A and frame C)

a) To solve for G_1 G_2 G_3 we must express $\vec{a}_z$ in terms of $\vec{b}_1$ $\vec{b}_2$ $\vec{b}_3$



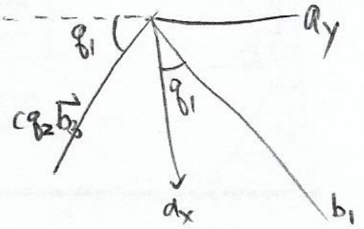
Note that $\vec{a}_z = cq_2 \vec{b}_2 + sq_2 \vec{b}_3$

$${}^{A}\vec{\omega}^{C} = -\dot{q}_2 \vec{b}_1 + (\dot{q}_1 cq_2)\vec{b}_2 + (\dot{q}_3 + \dot{q}_1 sq_2)\vec{b}_3$$

$G_2 = \dot{q}_2 = -U_1$ $\quad G_1 = \dot{q}_1 = \sec q_2 U_2$ $\quad G_3 = \dot{q}_3 = U_3 - G_1 sq_2 = U_3 - \tan q_2 U_2$

$$\therefore G_1 = U_2 \sec q_2 \;;\; G_2 = -U_1 \;;\; G_3 = U_3 - U_2 \tan q_2$$

b) To solve for F_1 F_2 F_3 we must express $\vec{b}_1$ and $\vec{b}_3$ in terms of $\vec{a}_1$ $\vec{a}_2$ $\vec{a}_3$



$$\vec{b}_1 = cq_1 \vec{a}_x + sq_1 \vec{a}_y$$

$$\vec{b}_3 = sq_1 cq_2 \vec{a}_x - cq_1 cq_2 \vec{a}_y + sq_2 \vec{a}_z$$

substituting $\vec{b}_1$ and $\vec{b}_3$ to ${}^{A}\vec{\omega}^{C}$

$$\begin{aligned} {}^{A}\vec{\omega}^{C} = \; & \dot{q}_3 sq_1 cq_2 \vec{a}_x - \dot{q}_3 cq_1 cq_2 \vec{a}_y + \dot{q}_3 sq_2 \vec{a}_z \\ & -\dot{q}_2 cq_1 \vec{a}_x - \dot{q}_2 sq_1 \vec{a}_y \\ & + \dot{q}_1 \vec{a}_z \end{aligned}$$

which gives us

$U_x = -cq_1 \dot{q}_2 + sq_1 cq_2 \dot{q}_3$ $\qquad U_y = -sq_1 \dot{q}_2 - cq_1 cq_2 \dot{q}_3$

$U_z = \dot{q}_1 + \dot{q}_3 sq_2$

Solve for $F_3 = \dot{q}_3$ from U_x and U_y equation

$$\frac{U_x}{cq_1} - \frac{U_y}{sq_1} = -\dot{q}_2 + \frac{sq_1}{cq_1} cq_2 \dot{q}_3 + \dot{q}_2 + \frac{cq_1}{sq_1} cq_2 \dot{q}_3 = \frac{sq_1^2 + cq_1^2}{sq_1 cq_1} cq_2 \dot{q}_3$$

$$\Rightarrow \dot{q}_3 = F_3 = (U_x sq_1 - U_y cq_1) \sec q_2$$

solve for F_2 : $U_x = -cq_1 F_2 + sq_1 cq_2 (U_x sq_1 - U_y cq_1) \sec q_2$ ~~// divide both sides~~

$$cq_1 F_2 = -U_x + sq_1^2 (U_x) - U_y sq_1 cq_1$$

$$F_2 = \frac{cq_1^2}{cq_1} U_x - \frac{sq_1 cq_1}{cq_1} U_y \Rightarrow F_2 = \dot{q}_2 = U_x cq_1 - U_y sq_1$$

solve for $F_1 = \dot{q}_1$: $\dot{q}_1 = U_z - \dot{q}_3 sq_2$

$$F_1 = \dot{q}_1 = U_z - (U_x sq_1 - U_y cq_1) \tan q_2$$

2.8 $^A\vec{\omega}^B = \dot{q}_1 \vec{a}_2 - \dot{q}_2 \vec{b}_1$

Following the sol'n from 2.7 we get

$^A\vec{\omega}^B = -\dot{q}_2 \vec{b}_1 + (\dot{q}_1 cq_2)\vec{b}_2 + (\dot{q}_1 sq_2)\vec{b}_3$

values obtained from 2.7 sol'n

~~to~~ Express $\dot{q}_1 sq_2$ in terms of $u_1\ u_2\ u_3$ where $u_1 = -\dot{q}_2 \quad u_2 = \dot{q}_1 cq_2$

$u_3 = \dot{q}_3 + \dot{q}_1 sq_2$

$$\dot{q}_1 sq_2 = \dot{q}_1 cq_2 \frac{sq_2}{cq_2} = u_2 \tan q_2$$

Hence

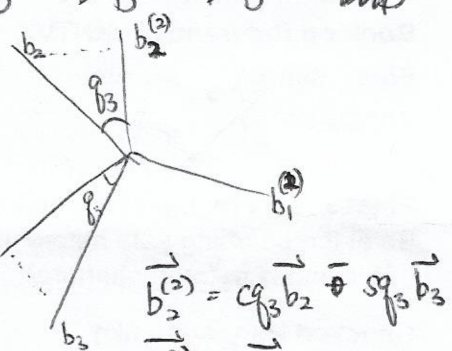
$$^A\vec{\omega}^B = u_1 \vec{b}_1 + u_2 \vec{b}_2 + u_2 \tan q_2\, \vec{b}_3$$

Q.9 ~~${}^A\vec{\omega}^B = \dot{q}_1 \vec{b}_1 + \dot{q}_2 \vec{b}_2 + \dot{q}_3 \vec{b}_3$~~ the frames in ... are not B frames

$${}^N\vec{\omega}^B = w_1 \vec{b}_1 + w_2 \vec{b}_2 + w_3 \vec{b}_3$$

$${}^N\vec{\omega}^A = \Omega \vec{a}_3$$

Let the transformation from frame A to frame B be $A \to B^{(1)} \to B^{(2)} \to B$



$${}^A\vec{\omega}^B = {}^A\vec{\omega}^{B^{(1)}} + {}^{B^{(1)}}\vec{\omega}^{B^{(2)}} + {}^{B^{(2)}}\vec{\omega}^{B}$$

$$= \dot{q}_1 \vec{a}_1 + \dot{q}_2 \vec{b}_2^{(2)} + \dot{q}_3 \vec{b}_1$$

$${}^N\vec{\omega}^B = {}^N\vec{\omega}^A + {}^A\vec{\omega}^B$$

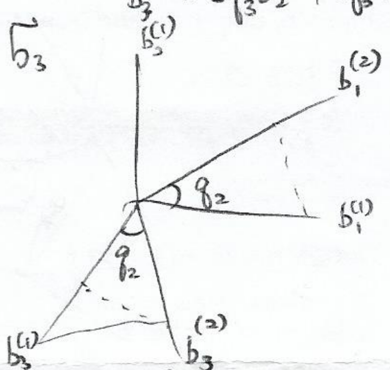
$$= \dot{q}_1 \vec{a}_1 + \dot{q}_2 \vec{b}_2^{(2)} + \dot{q}_3 \vec{b}_1 + \Omega \vec{a}_3$$

$\vec{b}_2^{(2)} = cq_3 \vec{b}_2 - sq_3 \vec{b}_3$

$\vec{b}_1^{(2)} = \vec{b}_1$

$\vec{b}_3^{(2)} = sq_3 \vec{b}_2 + cq_3 \vec{b}_3$

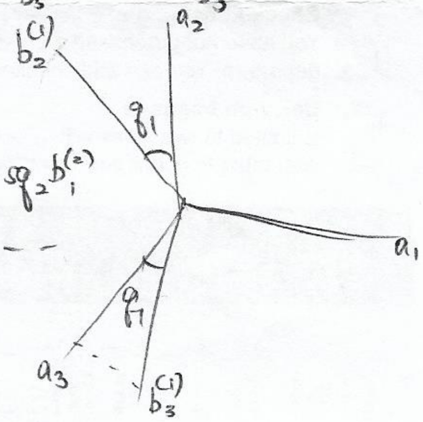
Now I must express $\vec{a}_1, \vec{a}_3$ and $\vec{b}_2^{(2)}$ in terms of $\vec{b}_1\ \vec{b}_2\ \vec{b}_3$



$$\vec{b}_2^{(2)} = cq_3 \vec{b}_2 - sq_3 \vec{b}_3$$

$$\vec{a}_1 = \vec{b}_1^{(1)} = cq_2 b_1^{(2)} + sq_2 b_3^{(2)}$$

$$= cq_2 \vec{b}_1 + sq_2 sq_3 \vec{b}_2 + sq_2 cq_3 \vec{b}_3$$



$$\vec{a}_3 = sq_1 b_2^{(1)} + cq_1 b_3^{(1)}$$

$b_2^{(1)} = b_2^{(2)}$

$b_3^{(1)} = cq_2 b_3^{(2)} - sq_2 b_1^{(2)}$

$$= sq_1 cq_3 \vec{b}_2 - sq_1 sq_3 \vec{b}_3 + cq_1 cq_2 b_3^{(2)} - cq_1 sq_2 b_1^{(2)}$$

$$= sq_1 cq_3 \vec{b}_2 - sq_1 sq_3 \vec{b}_3 + cq_1 cq_2 sq_3 \vec{b}_2 + cq_1 cq_2 cq_3 \vec{b}_3 - cq_1 sq_2 \vec{b}_1$$

$$\boxed{\begin{aligned} {}^N\vec{\omega}^B = &\left(\dot{q}_1 cq_2 + \dot{q}_3 - \Omega cq_1 sq_2\right)\vec{b}_1 + \\ &\left(\dot{q}_1 sq_2 sq_3 + \dot{q}_2 cq_3 + \Omega\left(sq_1 cq_3 + cq_1 cq_2 sq_3\right)\right)\vec{b}_2 + \\ &\left(\dot{q}_1 sq_2 cq_3 - \dot{q}_2 sq_3 + \Omega\left(-sq_1 sq_3 + cq_1 cq_2 cq_3\right)\right)\vec{b}_3 \end{aligned}}$$

2.10 $${}^A\vec{\alpha}^C = \frac{{}^A d\, {}^A\vec{\omega}^C}{dt} = \frac{{}^A d\left(u_x \vec{a}_x + u_y \vec{a}_y + u_z \vec{a}_z\right)}{dt}$$

$$\boxed{= \dot{u}_x \vec{a}_x + \dot{u}_y \vec{a}_y + \dot{u}_z \vec{a}_z}$$

$${}^A\vec{\alpha}^C = \frac{{}^A d\, {}^A\vec{\omega}^C}{dt} = \frac{{}^B d\, {}^A\vec{\omega}^C}{dt} + {}^A\vec{\omega}^B \times {}^A\vec{\omega}^C$$

Note that from prior sol'n

$${}^A\vec{\omega}^C = u_1 \vec{b}_1 + u_2 \vec{b}_2 + u_3 \vec{b}_3$$

$${}^A\vec{\omega}^B = u_1 \vec{b}_1 + u_2 \vec{b}_2 + u_2 \tan q_2 \vec{b}_3$$

$$= \dot{u}_1 \vec{b}_1 + \dot{u}_2 \vec{b}_2 + \dot{u}_3 \vec{b}_3$$

$$+ \left(u_2 u_3 - u_2^2 \tan q_2\right) \vec{b}_1 + \left(u_1 u_2 \tan q_2 - u_1 u_3\right) \vec{b}_2$$

$$\boxed{= \left(\dot{u}_1 + u_2\left(u_3 - u_2 \tan q_2\right)\right) \vec{b}_1 + \left(\dot{u}_2 + u_1\left(-u_3 + u_2 \tan q_2\right)\right) \vec{b}_2 + \dot{u}_3 \vec{b}_3}$$

2.11 $${}^L\vec{\omega}^C = \dot{q}_1 \vec{l}_x \Rightarrow {}^L\vec{\alpha}^C = \frac{{}^L d\, {}^L\vec{\omega}^C}{dt} = \ddot{q}_1 \vec{l}_x \Rightarrow \boxed{\left|{}^L\vec{\alpha}^C\right| = \left|\ddot{q}_1\right|}$$

$${}^C\vec{\omega}^R = \dot{q}_2 \vec{c}_2 \Rightarrow {}^C\vec{\alpha}^R = \frac{{}^C d\, {}^C\vec{\omega}^R}{dt} = \ddot{q}_2 \vec{c}_2 \Rightarrow \boxed{\left|{}^C\vec{\alpha}^R\right| = \left|\ddot{q}_2\right|}$$

$${}^L\vec{\omega}^R = {}^L\vec{\omega}^C + {}^C\vec{\omega}^R = \dot{q}_1 \vec{l}_x + \dot{q}_2 \vec{c}_2$$

Note: $\vec{c}_2 = -cq_1 \vec{l}_z + sq_1 \vec{l}_y$

$$= \dot{q}_1 \vec{l}_x + \dot{q}_2 sq_1 \vec{l}_y - \dot{q}_2 cq_1 \vec{l}_z$$

$$\frac{{}^L d\, {}^L\vec{\omega}^R}{dt} = {}^L\vec{\alpha}^R = \ddot{q}_1 \vec{l}_x + \left(\ddot{q}_2 sq_1 + \dot{q}_2 \dot{q}_1 cq_1\right) \vec{l}_y + \left(-\ddot{q}_2 cq_1 + \dot{q}_1 \dot{q}_2 sq_1\right) \vec{l}_z$$

$$\left|{}^L\vec{\alpha}^R\right| = \left(\ddot{q}_1^2 + \left(\ddot{q}_2 sq_1 + \dot{q}_1 \dot{q}_2 cq_1\right)^2 + \left(-\ddot{q}_2 cq_1 + \dot{q}_1 \dot{q}_2 sq_1\right)^2\right)^{1/2}$$

$$= \left(\ddot{q}_1^2 + \ddot{q}_2^2 sq_1^2 + 2\dot{q}_1 \dot{q}_2 \ddot{q}_2 sq_1 cq_1 + (\dot{q}_1 \dot{q}_2)^2 cq_1^2 + \ddot{q}_2^2 cq_1^2 - 2\dot{q}_1 \dot{q}_2 \ddot{q}_2 sq_1 cq_1 + (\dot{q}_1 \dot{q}_2)^2 sq_1^2\right)^{1/2}$$

$$\boxed{= \left(\ddot{q}_1^2 + \ddot{q}_2^2 + (\dot{q}_1 \dot{q}_2)^2\right)^{1/2}}$$

2.12

$^{A}\vec{\omega}^{B_3} = -6\vec{k}$ rad/s

$^{A}\vec{\alpha}^{B_1} = 5\vec{k}$ rad/s

Determine $^{A}\vec{\omega}^{B_1}$, $^{A}\vec{\omega}^{B_2}$, $^{A}\vec{\alpha}^{B_2}$, $^{A}\vec{\alpha}^{B_3}$ at time t^*

$\beta_1 = \frac{3}{5}\hat{\imath} + \frac{4}{5}\hat{\jmath}$ $\qquad \beta_1' = -\frac{4}{5}\hat{\imath} + \frac{3}{5}\hat{\jmath}$

$\beta_2 = -\hat{\imath}$ $\qquad \beta_2' = -\hat{\jmath}$

$\beta_3 = -\hat{\jmath}$ $\qquad \beta_3' = \hat{\imath}$

From eq $10\beta_1 + 9\beta_2 + 4\beta_3 + 5\beta_4 = 0$, derive wrt t

$\overset{⓪}{\Rightarrow} 10(\omega_1\vec{\beta_1'}) + 9\omega_2\vec{\beta_2'} + 4\omega_3\vec{\beta_3'} = 0$ ⇒ this gives us 2 eq.

$\overset{①}{\Rightarrow} -10\omega_1(\frac{4}{5})\hat{\imath} + 4\omega_3\hat{\imath} = 0$ ⇒ given $\omega_3 = -6$, $\omega_1 = -3$ rad/s

$\overset{②}{\Rightarrow} 10\omega_1(\frac{3}{5})\hat{\jmath} - 9\omega_2\hat{\jmath} = 0$ ⇒ $\omega_2 = -2$ rad/s

$^{A}\vec{\omega}^{B1} = -3\vec{k}$ rad/s $\qquad$ **$^{A}\vec{\omega}^{B_2} = -2\vec{k}$ rad/s**

Derive eq ⓪ wrt to t again

$10\alpha_1\vec{\beta_1'} + 10\omega_1^2(-\vec{\beta_1}) + 9\alpha_2\vec{\beta_2'} + 9\omega_2^2(-\vec{\beta_2}) + 4\alpha_3\vec{\beta_3'} + 4\omega_3^2(-\vec{\beta_3}) = 0$

$\overset{③\hat{\imath}}{\Rightarrow} 10\alpha_1(-\frac{4}{5}) - 10\omega_1^2(\frac{3}{5}) + 9\omega_2^2 + 4\alpha_3 = 0$ ⇒ $\alpha_3 = \frac{29}{2}$ rad/sec

$\overset{④\hat{\jmath}}{\Rightarrow} 10\alpha_1(\frac{3}{5}) - 10\omega_1^2(\frac{4}{5}) - 9\alpha_2 + 4\omega_3^2 = 0$ ⇒ $\alpha_2 = 34/3$ rad/s

$^{A}\vec{\alpha}^{B2} = 34/3\,\vec{k}$ rad/s $\qquad$ **$^{A}\vec{\alpha}^{B_3} = 29/2\,\vec{k}$ rad/s**