3.1 (a) reference frame L

$$^L\frac{\partial \vec{c}_1}{\partial t} = 0 \qquad ^L\frac{\partial \vec{c}_2}{\partial t} = \vec{c}_3\dot{q}_1 \qquad ^L\frac{\partial \vec{c}_3}{\partial t} = -\vec{c}_2\dot{q}_1$$

From 1.10 $\vec{p} = \vec{OC} + \vec{CP}$

$$= -3r\,cq_2\,\vec{c}_1 + r\,\vec{c}_2 + 3r\,sq_2\,\vec{c}_3$$

$$^L\frac{d\vec{p}}{dt} = r\left[\dot{q}_1\vec{c}_3 - 3\dot{q}_1\,sq_2\,\vec{c}_2 + 3\dot{q}_2\,cq_2\,\vec{c}_3 + 3\dot{q}_2\,sq_2\,\vec{c}_1\right]$$

$$\boxed{^L v^P = \,^L\frac{d\vec{p}}{dt} \overset{\text{rearrange}}{=} r\left[3sq_2\left(\dot{q}_2\vec{c}_1 - \dot{q}_1\vec{c}_2\right) + \left(\dot{q}_1 + 3\dot{q}_2 cq_2\right)\vec{c}_3\right]}$$

$$^L a^P = \frac{^A d\,^L v^P}{dt} = r\left[3cq_2\dot{q}_2\left(\dot{q}_2\vec{c}_1 - \dot{q}_1\vec{c}_2\right) + 3sq_2\left(\ddot{q}_2\vec{c}_1 - \cancel{\dot{q}_2 0} - \ddot{q}_1\vec{c}_2 - \dot{q}_1\dot{q}_1\vec{c}_3\right)\right.$$
$$\left. + \left(\ddot{q}_1 + 3\ddot{q}_2 cq_2 - 3\dot{q}_2 sq_2\right)\vec{c}_3 - \left(\dot{q}_1 + 3\dot{q}_2 cq_2\right)\dot{q}_1\vec{c}_2\right]$$

$$\boxed{\begin{aligned} = r\Big[\, & 3\left(cq_2\,\dot{q}_2^2 + sq_2\,\ddot{q}_2\right)\vec{c}_1 + \\ & \left(-6cq_2\dot{q}_1\dot{q}_2 - 3sq_2\ddot{q}_1 - \dot{q}_1^2\right)\vec{c}_2 + \\ & \left(-3sq_2\,\dot{q}_1^2 + \ddot{q}_1 + 3\ddot{q}_2 cq_2 - 3\dot{q}_2 sq_2\right)\vec{c}_3 \end{aligned}}$$

(b) reference frame C

$$\boxed{^C v^P = \frac{^C d\vec{P}}{dt} = 3r\,sq_2\dot{q}_2\,\vec{c}_1 + 0\,\vec{c}_2 + 3r\,cq_2\dot{q}_2\,\vec{c}_3}$$

$$\boxed{^C a^P = \frac{^C d\,^C v^P}{dt} = \left(3r(\ddot{q}_2 sq_2 + \dot{q}_2^2 cq_2\right)\vec{c}_1 + 3r\left(\ddot{q}_2 cq_2 - \dot{q}_2^2 sq_2\right)\vec{c}_3}$$

3.2 (a) show $\vec{V} = V\vec{b}_1$

$$\vec{V} = \cancel{\text{scalar velocity} \times \frac{d\vec{P}}{d\,\text{curve } C}}\ \frac{d\vec{P}}{dt} = \underbrace{\frac{d\vec{P}}{d\,\text{curve}C}}_{\vec{b}_1 \text{ or } p'} \cdot \underbrace{\frac{d\,\text{curve}C}{dt}}_{\text{scalar } V \text{ or } \dot{S}} = V\,\vec{b}_1 \quad \blacksquare$$

(b) $$\vec{a} = \frac{d\vec{V}}{dt} = \dot{V}\,\vec{b}_1 + V\frac{d\vec{b}_1}{dt}$$

$$= \boxed{\dot{V}\,\vec{b}_1 + \frac{V^2}{\rho}\vec{b}_2} \quad \blacksquare$$

Note: $\vec{b}_1 = p' \Rightarrow \frac{d\vec{b}_1}{dt} = \frac{d\vec{b}_1}{ds}\cdot\overset{V}{\frac{ds}{dt}}$

$= p''\,V$

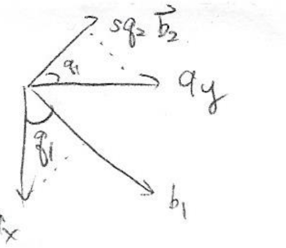
$\Rightarrow p'' = \vec{b}_2/\rho \Rightarrow \frac{d\vec{b}_1}{dt} = \vec{b}_2\frac{V}{\rho}$

3.3

$$^{A}\vec{p}^{\,C*} = q_4\,\vec{a}_x + q_5\,\vec{a}_y + R\,\vec{b}_2$$
$$= (q_4 - R\,sq_1 sq_2)\,\vec{a}_x + (q_5 + R\,cq_1 sq_2)\,\vec{a}_y + R\,cq_2\,\vec{a}_z$$

Note:

$$\vec{b}_1 = cq_1\,\vec{a}_x + sq_1\,\vec{a}_y$$
$$\vec{b}_2 = cq_2\,\vec{a}_z - sq_1 sq_2\,\vec{a}_x + cq_1 sq_2\,\vec{a}_y$$
$$\vec{b}_3 = sq_1 cq_2\,\vec{a}_x - cq_1 cq_2\,\vec{a}_y + sq_2\,\vec{a}_z$$



$$^{A}\vec{V}^{C*} = \frac{^{A}d\,^{A}\vec{p}^{\,C*}}{dt} = (\dot{q}_4 - R\dot{q}_1 cq_1 sq_2 - R\dot{q}_2 sq_1 cq_2)\,\vec{a}_x + (\dot{q}_5 - R\dot{q}_1 sq_1 sq_2 + R\dot{q}_2 cq_1 cq_2)\,\vec{a}_y - R\dot{q}_2 sq_2\,\vec{a}_z$$

$$V_1 = {}^{A}\vec{V}^{C*}\cdot\vec{b}_1 = \dot{q}_4 cq_1 - R\dot{q}_1 cq_1^2 sq_2 + \dot{q}_5 sq_1 - R\dot{q}_1 sq_1^2 sq_2$$

Note $u_4 = \dot{q}_4$, $u_5 = \dot{q}_5$, $u_2 = \dot{q}_1 \cos q_2$, $u_1 = -\dot{q}_2$

$$\boxed{V_1 = u_4 cq_1 - R u_2 \tan q_2 + u_5 sq_1}$$

$$V_2 = {}^{A}\vec{V}^{C*}\cdot\vec{b}_2 = -\dot{q}_4 sq_1 sq_2 + \cancel{R\dot{q}_2 sq_1^2 sq_2 cq_2} + \dot{q}_5 cq_1 sq_2 + \cancel{R\dot{q}_2 cq_1^2 sq_2 cq_2} - \cancel{R\dot{q}_2 sq_2 cq_2}$$

$$\boxed{V_2 = -u_4 sq_1 sq_2 + u_5 cq_1 sq_2}$$

$$V_3 = {}^{A}\vec{V}^{C*}\cdot\vec{b}_3 = \dot{q}_4 sq_1 cq_2 - R\dot{q}_2 sq_1^2 cq_2^2 - \dot{q}_5 cq_1 cq_2 - R\dot{q}_2 cq_1^2 cq_2^2 - R\dot{q}_2 sq_2^2$$

$$\boxed{V_3 = u_4 sq_1 cq_2 - u_5 cq_1 \cos q_2 + R u_1}$$

3.4

$$^{A}\vec{a}^{C*} = \frac{^{A}d\,^{A}\vec{V}^{C*}}{dt} = (\ddot{q}_4 - R\ddot{q}_1 cq_1 sq_2 + R\dot{q}_1^2 sq_1 sq_2 - 2R\dot{q}_1\dot{q}_2 cq_1 cq_2 - R\ddot{q}_2 sq_1 cq_2 \cancel{-R\dot{q}_1\dot{q}_2 cq_1 cq_2} + R\dot{q}_2^2 sq_1 sq_2)\,\vec{a}_x + (\ddot{q}_5 - R\ddot{q}_1 sq_1 sq_2 - R\dot{q}_1^2 cq_1 sq_2 - 2R\dot{q}_1\dot{q}_2 sq_1 cq_2 + R\ddot{q}_2 cq_1 cq_2 \cancel{-R\dot{q}_1\dot{q}_2 sq_1 cq_2} - R\dot{q}_2^2 cq_1 sq_2)\,\vec{a}_y + (-R\ddot{q}_2 sq_2 - R\dot{q}_2^2 cq_2)\,\vec{a}_z$$

Note:

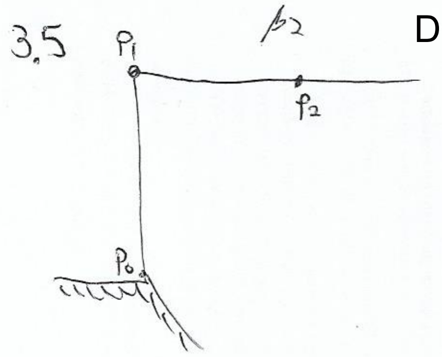
$$\dot{u}_2 = \ddot{q}_1 cq_2 - \dot{q}_1\dot{q}_2 sq_2$$
$$sq_2\ddot{q}_1 = sq_2\frac{\dot{u}_2}{cq_2} + \frac{\dot{q}_1\dot{q}_2 sq_2^2}{cq_2^2}cq_2$$
$$R\dot{u}_2 \tan q_2 + R u_1 u_2 \tan^2 q_2$$

$$a_1 = {}^{A}\vec{a}^{C*}\cdot\vec{b}_1 = \ddot{q}_4 cq_1 - R\ddot{q}_1 sq_2 - 2R\dot{q}_1\dot{q}_2 cq_2 + \ddot{q}_5 sq_1$$
$$= \dot{u}_4 cq_1 - R\dot{u}_2\tan q_2 + R u_1 u_2 \tan^2 q_2 + 2R u_1 u_2 + \dot{u}_5 sq_1$$
$$\boxed{= \dot{u}_4 cq_1 - R\dot{u}_2\tan q_2 + R u_1 u_2 (1 + \sec^2 q_2) + \dot{u}_5 sq_1}$$

$$a_2 = {}^{A}\vec{a}^{C*}\cdot\vec{b}_2 = -\ddot{q}_4 sq_1 sq_2 + \ddot{q}_5 cq_1 sq_2 - R\dot{q}_1^2 sq_2^2 + \cancel{R\ddot{q}_2 sq_2 cq_2} - R\dot{q}_2^2 \cancel{sq_2^2} - \cancel{R\ddot{q}_2 sq_2 cq_2} - \cancel{R\dot{q}_2^2 cq_2^2}$$
$$\boxed{= (\dot{u}_5 cq_1 - \dot{u}_4 sq_1)\,sq_2 - R u_2^2 \tan^2 q_2 - R u_1^2}$$

$$a_3 = {}^{A}\vec{a}^{C*}\cdot\vec{b}_3 = \ddot{q}_4 sq_1 cq_2 - \ddot{q}_5 cq_1 cq_2 + R\dot{q}_1^2 sq_2 cq_2 - R\ddot{q}_2 \cancel{cq_2^2} + \cancel{R\dot{q}_2^2 sq_2 cq_2} - \cancel{R\ddot{q}_2 sq_2^2} - \cancel{R\dot{q}_2^2 sq_2 cq_2}$$
$$\boxed{= (\dot{u}_4 sq_1 - \dot{u}_5 cq_1)\,cq_2 + R u_2^2 \tan q_2 + R\dot{u}_1}$$

3.5



$^A v^{P_1} = \cancel{^A v^{P_0}} + {}^A\vec{\omega}^{\beta_3} \times (\beta_3 * 4)$

$$^A v^{P_2} = {}^A v^{P_1} + {}^A\vec{\omega}^{\beta_2} \times \left(\tfrac{0.5*}{-\beta_2} * 9\right)$$
$$= {}^A\vec{\omega}^{\beta_3} \times (4\hat{j}) + {}^A\vec{\omega}^{\beta_2} \times \left(\tfrac{9}{2}\hat{i}\right)$$
$$= 24\hat{i} - 9\hat{j} = \boxed{-24\vec{\beta_2} + 9\vec{\beta_3}}$$

Note: from 2.12

β_1

$\vec{\beta_2} = -\hat{i}$

$\vec{\beta_3} = -\hat{j}$

${}^A\vec{\omega}^{\beta_3} = -6\vec{K}$

${}^A\vec{\omega}^{\beta_2} = -2\vec{K}$

${}^A\alpha^{\beta_3} = 29/2\,\vec{K}$

${}^A\alpha^{\beta_2} = 34/3\,\vec{K}$

$$^A a^{P_1} = \cancel{^A a^{P_0}} + {}^A\vec{\omega}^{\beta_3} \times \left({}^A\vec{\omega}^{\beta_3} \times (-4\beta_3)\right) + {}^A\vec{\alpha}^{\beta_3} \times (-4\beta_3)$$
$$= -6\vec{K} \times (-6\vec{K} \times 4\hat{j}) + \tfrac{29}{2}\vec{K} \times 4\hat{j}$$
$$= -144\hat{j} - 58\hat{i}$$

$$^A a^{P_2} = {}^A a^{P_1} + {}^A\vec{\omega}^{\beta_2} \times \left({}^A\vec{\omega}^{\beta_2} \times \left(-\tfrac{9}{2}\beta_2\right)\right) + {}^A\vec{\alpha}^{\beta_2} \times \left(-\tfrac{9}{2}\beta_2\right)$$
$$= -144\hat{j} - 58\hat{i} + -2\vec{K} \times \left(-2\vec{K} \times \tfrac{9}{2}\hat{i}\right) + \tfrac{34}{3}\vec{K} \times \tfrac{9}{2}\hat{i}$$
$$= -144\hat{j} - 58\hat{i} - 18\hat{i} + 51\hat{j} = -93\hat{j} - 76\hat{i}$$
$$\boxed{= 76\vec{\beta_2} + 93\vec{\beta_3}}$$

3.6

$$^A\vec{v}^{\hat{C}} = {}^A\vec{v}^{C*} + {}^A\vec{\omega}^C \times (-R\vec{b_2}) = {}^A\vec{v}^{C*} + (Ru_3\vec{b_1} - Ru_1\vec{b_3})$$

$\begin{vmatrix} u_1 & u_2 & u_3 \\ 0 & -R & 0 \end{vmatrix}$

Note: from 2.7: $\vec{b_1} = cq_1\vec{a_x} + sq_1\vec{a_y}$

$\vec{b_3} = sq_1cq_2\vec{a_x} - cq_1cq_2\vec{a_y} + sq_2\vec{a_z}$

$$= {}^A\vec{v}^{C*} + (Ru_3cq_1 + R\dot{q}_2sq_1cq_2)\vec{a_x} + (Ru_3sq_1 - R\dot{q}_2cq_1cq_2)\vec{a_y} + R\dot{q}_2sq_2\vec{a_z}$$

refer to 3.3 soln

$$= (\dot{q}_4 - R\dot{q}_1cq_1sq_2 + Ru_3cq_1)\vec{a_x} + (\dot{q}_5 - R\dot{q}_1sq_1sq_2 + Ru_3sq_1)\vec{a_y}$$

$\vec{u_2} = \dot{q}_1cq_2$

$\vec{u_4} = \dot{q}_4$ $\vec{u_5} = \dot{q}_5$

$$^A\vec{v}^{\hat{C}} \boxed{= (u_4 + Rcq_1(u_3 - u_2\tan q_2))\vec{a_x} + (u_5 + Rsq_1(u_3 - u_2\tan q_2))\vec{a_y}}$$

$$^A\vec{a}^{\hat{C}} = {}^A\vec{a}^{C*} + {}^A\vec{\omega}^C \times ({}^A\vec{\omega}^C \times \overset{-R\vec{b_2}}{\vec{r}}) + {}^A\vec{\alpha}^C \times \overset{-R\vec{b_2}}{\vec{r}} = {}^A\vec{a}^{C*} + {}^A\vec{\omega}^C \times (Ru_3\vec{b_1} - Ru_1\vec{b_3}) + {}^A\vec{\alpha}^C \times (-R\vec{b_2})$$

$\begin{vmatrix} u_1 & u_2 & u_3 \\ Ru_3 & 0 & -Ru_1 \end{vmatrix}$

$\begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & -R & 0 \end{vmatrix}$

$R\alpha_3\vec{b_1} - R\alpha_1\vec{b_3}$

refer to

$$= {}^A\vec{a}^{C*} + (-Ru_1u_2\vec{b_1} + (Ru_3^2 + Ru_1^2)\vec{b_2} - Ru_2u_3\vec{b_3})$$
$$+ R\dot{u}_3\vec{b_1} - R(\dot{u}_1 + u_2(u_3 - u_2\tan q_2))\vec{b_3}$$

$$\boxed{\begin{aligned} {}^A\vec{a}^{\hat{C}} = &(\dot{u}_4cq_1 - R\dot{u}_2\tan q_2 + Ru_1u_2\sec^2 q_2 + \dot{u}_5sq_1 + R\dot{u}_3)\vec{b_1} + \\ &((\dot{u}_5cq_1 - \dot{u}_4sq_1)sq_2 + R(u_3^2 - u_2^2\tan^2 q_2))\vec{b_2} + \\ &((\dot{u}_4sq_1 - \dot{u}_5cq_1)cq_2 - 2Ru_2(u_3 - u_2\tan q_2))\vec{b_3} \end{aligned}}$$

3.7 $${}^A\vec{a}^{\hat{C}} \overset{2.6.2}{=} \frac{{}^A d\,{}^A\vec{v}^{\hat{C}}}{dt} = \frac{dv_x}{dt}\vec{a}_x + \frac{dv_y}{dt}\vec{a}_y \quad \text{(by definition)}$$

$$\bar{a}_i = {}^A\vec{a}^{\hat{C}} \cdot b_i \qquad \therefore\ \bar{a}_i == \hat{a}_i$$

3.8 By definition, plane H is fixed in a ref. frame A. Let $\hat{H}$ be part of plane H that is in contact w/ the rigid body C. Since C is rolling on plane H, by definition of rolling we have ${}^A V^{\hat{H}} = {}^A V^{\hat{C}}$. Since plane H is fixed in frame A, ${}^A V^{\hat{H}} = 0$.

Refer to sol'n of ${}^A V^{\hat{C}}$ from 3.6

$$\boxed{\begin{array}{l} \text{from } \vec{a}_x \text{ component}: \quad u_4 = Rcq_1(u_2 \tan q_2 - u_3) \\ \text{from } \vec{a}_y \text{ component}: \quad u_5 = Rsq_1(u_2 \tan q_2 - u_3) \end{array}}$$

3.8 Let $\hat{H}$ be part of plane H that is in contact w/ the rigid body C

Since C rolls on plane H, ${}^A v^{\hat{H}} = {}^A v^{\hat{C}}$. Since plane H is fixed in frame A, ${}^A v^{\hat{H}} = 0$

Referring to sol'n of ${}^A v^{\hat{c}}$ from 3.6, ${}^A v^{\hat{c}} = 0$

$$\boxed{\begin{aligned} &\text{from } \vec{a_x} \text{ component:} \quad u_4 = R\,cq_1 (u_2 \tan q_2 - u_3) \\ &\text{from } \vec{a_y} \text{ component:} \quad u_5 = R\,sq_1 (u_2 \tan q_2 - u_3) \end{aligned}}$$

3.9 Note: $u_1 = -\dot{q}_2 \qquad u_2 = \dot{q}_1 cq_2 \qquad u_3 = \dot{q}_3 + \dot{q}_1 sq_2$ from 2.7

$u_4 = -R cq_1 \dot{q}_3 \qquad \dot{u}_4 = R\dot{q}_1\dot{q}_3 sq_1 - R\ddot{q}_3 cq_1$

$u_5 = -R sq_1 \dot{q}_3 \qquad \dot{u}_5 = -R\dot{q}_1\dot{q}_3 cq_1 - R\ddot{q}_3 sq_1$

$\dot{u}_2 = \ddot{q}_1 cq_2 - \dot{q}_1\dot{q}_2 sq_2 \qquad \dot{u}_3 = \ddot{q}_3 + \ddot{q}_1 sq_2 + \dot{q}_1\dot{q}_2 cq_2$

Substituting these to ${}^A\vec{a}^{\hat{c}}$ (3.6)

$${}^A\vec{a}^{\hat{c}} = \Big(\underbrace{\cancel{-R\ddot{q}_3}}_{\text{from } \dot{u}_4 \text{ and } u_5} - \cancel{R\ddot{q}_1 sq_2} + R\dot{q}_1\dot{q}_2 sq_2 \tan q_2 + R(-\dot{q}_2)\dot{q}_1 \frac{\cancel{cq_2}}{cq_2^{\cancel{2}}} + \cancel{R\ddot{q}_3} + \cancel{R\ddot{q}_1 sq_2} + R\dot{q}_1\dot{q}_2 cq_2\Big)\vec{b_1}$$

$$\Big(\cancel{-R\dot{q}_1\dot{q}_3 sq_2} + R\big(\dot{q}_3 + \cancel{\dot{q}_1 sq_2 - \dot{q}_1 sq_2}\big)\big(\dot{q}_3 + \dot{q}_1 sq_2 + \cancel{\dot{q}_1 sq_2}\big)\Big)\vec{b_2} +$$

$$\Big(R\dot{q}_1\dot{q}_3 cq_2 - 2R\dot{q}_1 cq_2\big(\dot{q}_3 + \cancel{\dot{q}_1 sq_2 - \dot{q}_1 sq_2}\big)\Big)\vec{b_3}$$

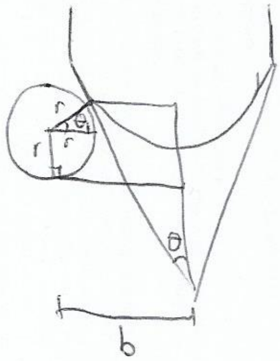
$$= R\dot{q}_1\dot{q}_2 \big(\underbrace{\cancel{sq_2 \tan q_2 - \sec q_2 + cq_2}}_{\frac{sq_2^2 - 1 + cq_2^2}{cq_2}}\big)\vec{b_1} + \big(R\dot{q}_3^2 + R\dot{q}_3\dot{q}_1 sq_2\big)\vec{b_2} - R\dot{q}_1\dot{q}_3 cq_2 \vec{b_3}$$

0

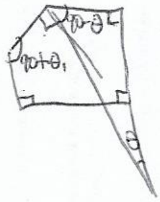
$$= R\dot{q}_3\Big[\big(\dot{q}_3 + \dot{q}_1 sq_2\big)\vec{b_2} - \dot{q}_1 cq_2 \vec{b_3}\Big] = R\dot{q}_3\Big[u_3\vec{b_2} - u_2\vec{b_3}\Big]$$

$$\boxed{\left|{}^A\vec{a}^{\hat{c}}\right| = R\,|\dot{q}_3|\,\left(u_3^2 + u_2^2\right)^{1/2}}$$

3.10 Attempt



First, let us find θ_1.

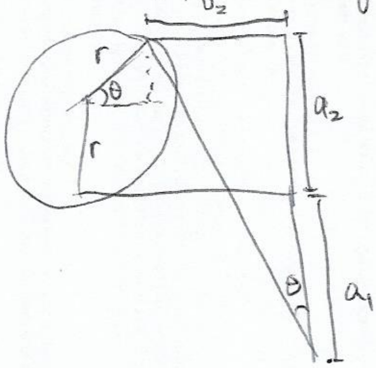


must have a total of 180 * 3 degrees

$$6+90-\theta_1+\theta = 180 * 3$$

$$\Rightarrow \theta_1 = \theta$$

Second, find an equation for b. Suppose the ~~contact between~~ distance between C and R contact to the cone vectex is "a_1". and a_2 and b_1 are as labeled (see image to the left)



$$\tan\theta = \frac{b_2}{a_1 + a_2}$$

$$b_2 = b - r\cos\theta$$
$$a_2 = r(1+\sin\theta)$$
} geometry from observing image to the left

$$\frac{\sin\theta}{\cos\theta} = \frac{b - r\cos\theta}{a_1 + r(1+\sin\theta)}$$

$$a_1\sin\theta + r(\sin\theta + \sin^2\theta) = b\cos\theta - r\cos^2\theta$$

$$r(\sin\theta + 1) = b\cos\theta - a_1\sin\theta \qquad \text{if } a_1 == b$$

$$\Rightarrow b = r(\sin\theta + 1)/(\cos\theta - \sin\theta)$$

which similar to the answer key given.
However, I don't understand why $a_1 == b$...

3.11 Show that ${}^F\omega^D = \frac{a}{2d}\left({}^F\omega^A + {}^F\omega^{A'}\right)$

As described by the problem

$\Rightarrow {}^F\omega^A = {}^F\omega^C + {}^C\omega^B$ ①

$\Rightarrow R\,{}^C\omega^B = r\,{}^C\omega^b \Rightarrow {}^C\omega^B = \frac{r}{R}\,{}^C\omega^b$ ③

similarly

$\Rightarrow {}^F\omega^{A'} = {}^F\omega^C + {}^C\omega^{B'}$ ②

Ensuring rolling at P' gives us

$$\left.\begin{aligned} &\Rightarrow {}^C v^{\hat{B}'} = {}^C\vec{\omega}^{B'} \times (-R\vec{N}) = -R\,{}^C\omega^{B'}(\vec{N}\times\vec{n}) \\ &{}^C v^{\hat{b}_2} = {}^C\vec{\omega}^{b} \times (-r\vec{N}) = -r\,{}^C\omega^{b}(n\times\vec{N}) \end{aligned}\right\} \quad \begin{aligned} &-R\,{}^C\omega^{B'} = r\,{}^C\omega^b \\ &\Rightarrow {}^C\omega^{B'} = -\frac{r}{R}\,{}^C\omega^b \text{ ④} \end{aligned}$$

Ensuring rolling at Q gives us

$$\left.\begin{aligned} &\Rightarrow {}^F v^{\hat{G}} = {}^F\vec{\omega}^{G} \times (-d\vec{N}) = -d\,{}^F\omega^{G}(\vec{n}\times\vec{N}) \\ &{}^F v^{\hat{E}} = {}^F\vec{\omega}^{E} \times (a\vec{n}) = a\,{}^F\omega^{E}(\vec{N}\times\vec{n}) \end{aligned}\right\} \quad \begin{aligned} &d\,{}^F\omega^G = a\,{}^F\omega^E \\ &\Rightarrow {}^F\omega^E = {}^F\omega^C = \frac{d}{a}\,{}^F\omega^G \end{aligned}$$

since C is fixed to E ${}^F\omega^C = {}^F\omega^E$

Since D is fixed to G, ${}^F\omega^G = {}^F\omega^D$. $\Rightarrow {}^F\omega^C = \frac{d}{a}\,{}^F\omega^D$ ⑤

Returning to ${}^F\omega^D = \frac{a}{2d}\left({}^F\omega^A + {}^F\omega^{A'}\right)$

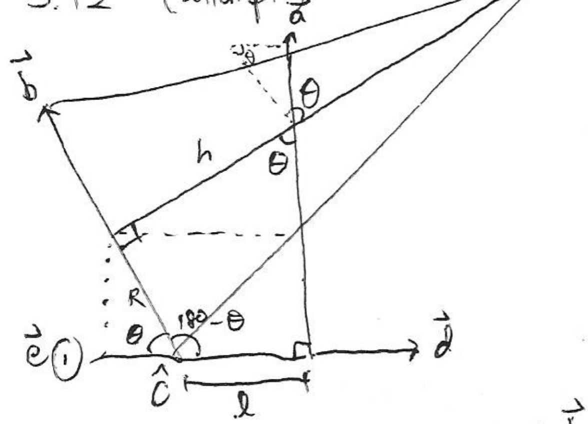
$$= \frac{a}{2d}\left(\underbrace{{}^F\omega^C + {}^C\omega^B}_{①} + \underbrace{{}^F\omega^C + {}^C\omega^{B'}}_{②}\right)$$

$$= \frac{a}{\cancel{2}d}\left(\cancel{2}\,\underbrace{{}^F\omega^C}_{⑤} + \cancel{\underbrace{\frac{r}{R}\,{}^C\omega^b}_{③}} - \cancel{\underbrace{\frac{r}{R}\,{}^C\omega^b}_{④}}\right)$$

$$= \frac{\cancel{a}}{\cancel{d}}\left(\frac{\cancel{d}}{\cancel{a}}\,{}^F\omega^D\right)$$

${}^F\omega^D = {}^F\omega^D$ ∎

3.12 (attempt)



$\vec{d} = \sin\theta\,\vec{c} - \cos\theta\,\vec{b}$

$\vec{a} = \cos\theta\,\vec{c} + \sin\theta\,\vec{b}$

$l = h s\theta - R c\theta$

$${}^A\vec{V}^{\hat{c}} = {}^A\vec{\omega}^a \times -l\vec{d} \qquad \begin{vmatrix} \vec{b} & \vec{c} & \vec{e} \\ \Omega s\theta & \Omega c\theta & 0 \\ +l c\theta & -l s\theta & 0 \end{vmatrix}$$

$$\left(-\Omega l s\theta^2 - \Omega l c\theta^2\right)\vec{e}$$

$$= -\Omega(h s\theta - R c\theta)\,\vec{e}$$

$${}^A\vec{V}^{\hat{c}} = {}^A\vec{V}^{c^*} + {}^A\vec{\omega}^c \times \vec{r} \qquad \begin{vmatrix} \vec{b} & \vec{c} & \vec{e} \\ \Omega s\theta & \Omega s\theta \frac{h}{R} & 0 \\ -R & -h & 0 \end{vmatrix}$$

$$= 0 + -h\Omega s\theta + R s\theta\,\Omega\frac{h}{R}$$

$$= 0$$

why is this zero... that I do not understand

3.13

$$^{B}\vec{r}^{P} = c(\Omega^2 t^2 - 1)\,\vec{b}_3$$

what is $^{N}\vec{a}^{P}$ for $t = 1/\Omega$, $q_1 = q_2 = q_3 = \pi/2$?

① $^{N}\vec{a}^{P} = {}^{N}\vec{a}^{\bar{B}} + {}^{B}\vec{a}^{P} + 2\,{}^{N}\vec{\omega}^{B} \times {}^{B}\vec{v}^{P}$

② $\dfrac{d\,{}^{B}\vec{r}^{P}}{dt} = {}^{B}\vec{v}^{P} = 2c\Omega^2 t\,\vec{b}_3$

③ $\dfrac{d\,{}^{B}\vec{v}^{P}}{dt} = {}^{B}\vec{a}^{P} = 2c\Omega^2\,\vec{b}_3$

④ @ $q_1 = q_2 = q_3 = \pi/2$ or $90°$

$$^{N}\omega^{B}_{0} = (\dot{q}_3)\vec{b}_1 + (\dot{q}_1)\,\vec{b}_2 + (-\dot{q}_2)\,\vec{b}_3$$

$$^{N}\omega^{B} \times {}^{B}v^{P} = \begin{vmatrix} \dot{q}_3 & \dot{q}_1 & -\dot{q}_2 \\ 0 & 0 & 2c\Omega^2 t \end{vmatrix}$$

$$= 2\dot{q}_1 c\Omega^2 t\,\vec{b}_1 - 2c\Omega^2 t\,\dot{q}_3\,\vec{b}_2$$

(4 cont.) $2\,{}^{N}\vec{\omega}^{B} \times {}^{B}\vec{v}^{P} = 4\dot{q}_1 c\Omega^2 t\,\vec{b}_1 - 4\dot{q}_3 c\Omega^2 t\,\vec{b}_2$

@ $t = 1/\Omega \Rightarrow 4\dot{q}_1 c\Omega\,\vec{b}_1 - 4\dot{q}_3 c\Omega\,\vec{b}_2$

⑤ 2.1.2 2pts on a rigid body

$$^{N}\vec{a}^{\bar{B}} = {}^{N}\vec{a}^{O} + {}^{N}\vec{\omega}^{B} \times ({}^{N}\vec{\omega}^{B} \times \vec{r}_1) + {}^{A}\vec{\alpha}^{B} \times \vec{r}_1$$

$^{N}\vec{\omega}^{B} = \Omega\,\vec{a}_3$ $\quad {}^{A}\vec{\alpha}^{B} = 0$ $\quad \vec{r}_1 = R\vec{a}_1 + c(\Omega^2 t^2 - 1)\,\vec{b}_3$

$^{N}\vec{a}^{O} = 0$ $\quad$ note that @ $t = \frac{1}{\Omega}$, $\vec{r}_1 = R\vec{a}_1$

$$= 0 + \Omega\vec{a}_3 \times (\Omega\vec{a}_3 \times R\vec{a}_1) + 0$$

$$= \Omega\vec{a}_3 \times R\Omega\vec{a}_2 = -R\Omega^2\,\vec{a}_1$$

$$= -R\Omega^2\,\vec{b}_2$$

Note: $\vec{a}_1 = cq_2\,\vec{b}_1 + sq_2 sq_3\,\vec{b}_2 + sq_3 cq_3\,\vec{b}_3$ from 2.9 sol'n

@ $t = \frac{1}{\Omega}$

$\vec{a}_1 = \vec{b}_2$

Combining ① to ⑤

$$\boxed{{}^{N}\vec{a}^{P} = 4\dot{q}_1 c\Omega\,\vec{b}_1 - (4\dot{q}_3 c\Omega + R\Omega^2)\,\vec{b}_2 + 2c\Omega^2\,\vec{b}_3}$$

3.14

$${}^A\vec{p}^{\,P} = q_4\,\vec{a_x} + q_5\,\vec{a_y}$$

$${}^A\vec{V}^P = \frac{d\,{}^A\vec{p}^{\,P}}{dt} = \dot{q}_4\,\vec{a_x} + \dot{q}_5\,\vec{a_y}$$

$$\boxed{{}^A\vec{V}^P = u_4\,\vec{a_x} + u_5\,\vec{a_y} + 0\,\vec{a_z}}$$

Note
$u_4 = \dot{q}_4$
$u_5 = \dot{q}_5$

Eq. 2.8.1

$${}^A\vec{V}^P = {}^A\vec{V}^{\hat{C}} + {}^C\vec{V}^P$$

$$\Rightarrow {}^C\vec{V}^P = {}^A\vec{V}^P - \underbrace{{}^A\vec{V}^{\hat{C}}}_{\text{from 3.6 sol'n}}$$

convert to $\vec{b}$ vectors $= -Rcq_1(u_3 - u_2\tan q_2)\,\vec{a_x} + -Rsq_1(u_3 - u_2\tan q_2)\,\vec{a_y}$

$$\boxed{{}^C\vec{V}^P = R(u_2\tan q_2 - u_3)\,\vec{b_1} + 0\,\vec{b_2} + 0\,\vec{b_3}}$$

Note:
$\vec{b_1} = cq_1\,\vec{a_x} + sq_1\,\vec{a_y}$

$+ (u_4 sq_1 cq_2 - u_5 cq_1 cq_2)\,\vec{b_3}$
$+ (-u_4 sq_1 sq_2 + u_5 cq_1 sq_2)\,\vec{b_2}$
$= (u_4 cq_1 + u_5 sq_1)\,\vec{b_1}$

Eq 2.8.2

$${}^A\vec{a}^P = {}^A\vec{a}^{\hat{C}} + {}^C\vec{a}^P + 2\,{}^A\vec{\omega}^C \times {}^C\vec{V}^P$$

$$\Rightarrow {}^C\vec{a}^P = {}^A\vec{a}^P - {}^A\vec{a}^{\hat{C}} - 2\,{}^A\vec{\omega}^C \times {}^C\vec{V}^P$$

$$= \left(R\dot{u}_2\tan q_2 - Ru_1u_2\sec^2 q_2 - R\dot{u}_3\right)\vec{b_1} +$$

$$\left(R(u_2\tan q_2 - u_3)(u_2\tan q_2 + u_3) - R(u_2\tan q_2 - u_3)\,2u_3\right)\vec{b_2} +$$

$$\left(2Ru_2(u_3 - u_2\tan q_2) + 2u_2R(u_2\tan q_2 - u_3)\right)\vec{b_3}$$

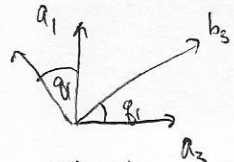
$$\boxed{\begin{aligned}{}^C\vec{a}^P = {} & \left(R\dot{u}_2\tan q_2 - Ru_1u_2\sec^2 q_2 - R\dot{u}_3\right)\vec{b_1} + \\ & \left(R(u_2\tan q_2 - u_3)^2\right)\vec{b_2} + 0\,\vec{b_3}\end{aligned}}$$

Note:
$${}^A\vec{a}^P = \frac{d\,{}^A\vec{V}^P}{dt} = \dot{u}_4\,\vec{a_x} + \dot{u}_5\,\vec{a_y}$$

${}^A\vec{a}^{\hat{C}}$ see prob 3.6 sol'n

$${}^A\vec{\omega}^C \times {}^C\vec{V}^P = \begin{vmatrix} u_1 & u_2 & u_3 \\ R(u_2\tan q_2 - u_3) & 0 & 0 \end{vmatrix}$$

$$= u_3R(u_2\tan q_2 - u_3)\,\vec{b_2} - u_2R(u_2\tan q_2 - u_3)\,\vec{b_3}$$



3.15 $\boxed{{}^E\vec{\omega}^A = u_1 \vec{a}_1}$

$${}^E\vec\omega^B = {}^E\vec\omega^A + {}^A\vec\omega^B = u_1\vec a_1 + u_2 \vec b_2$$
$$= \boxed{u_1 cq_1 \vec b_1 + u_2\vec b_2 + u_1 sq_1 \vec b_3}$$

$\vec a_1 = \vec b_1 cq_1 + \vec b_3 sq_1$
$\vec a_3 = \vec b_1 sq_1 + \vec b_3 cq_1$
$\vec a_2 = \vec b_2$
$u_2 = \dot q_1$

$$\boxed{{}^E\vec\omega^C = {}^E\vec\omega^D = {}^E\vec\omega^B}$$

$${}^E\vec\alpha^A = \frac{d\,{}^E\vec\omega^A}{dt} = \dot u_1 \vec a_1$$

$${}^E\vec\alpha^B = {}^E\vec\alpha^C = {}^E\vec\alpha^D = {}^E\vec\alpha^B = \frac{{}^E d\,{}^E\vec\omega^B}{dt} = \frac{{}^B d\,{}^E\vec\omega^B}{dt} = \boxed{(\dot u_1 cq_1 \mp u_1 u_2 sq_1)\vec b_1 + \dot u_2 \vec b_2 + (\dot u_1 sq_1 + u_1 u_2 cq_1)\vec b_3}$$

$$\overset{27.1}{{}^E\vec v^{A*}} = {}^E\vec\omega^A \times L_A \vec a_3 = \begin{vmatrix} u_1 & 0 & 0 \\ 0 & 0 & L_A \end{vmatrix} = -L_A u_1 \vec a_2 = \boxed{-L_A u_1 \vec b_2}$$

$${}^E\vec v^{B*} = {}^E\vec v^P + {}^E\vec\omega^B \times L_B \vec b_3 = -L_P u_1 \vec b_2 + \begin{vmatrix} u_1 cq_1 & u_2 & u_1 sq_1 \\ 0 & 0 & L_B \end{vmatrix} = -L_P u_1 \vec b_2 + u_2 L_B \vec b_1 - L_B u_1 cq_1 \vec b_2$$
$$= \boxed{u_2 L_B \vec b_1 - (L_P u_1 + L_B u_1 cq_1)\vec b_2}$$

$${}^E\vec v^{C*} = \underbrace{{}^E\vec v^{\bar B}}_{{}^E\vec v^{B*} \text{ but } l = L_B + q_2} + {}^B\vec v^{C*} = \boxed{u_2 (L_B + q_2)\vec b_1 - u_1(L_P + (L_B + q_2) cq_1)\vec b_2 + u_3 \vec b_3}$$

$${}^E\vec v^{D*} = {}^E\vec v^{C*} + {}^E\vec\omega^D \times (p_1 \vec b_1 + p_2 \vec b_2 + p_3 \vec b_3)$$
$$\begin{vmatrix} u_1 cq_1 & u_2 & u_1 sq_1 \\ p_1 & p_2 & p_3 \end{vmatrix} = (u_2 p_3 - u_1 p_2 sq_1)\vec b_1 + (u_1 p_1 sq_1 - u_1 p_3 cq_1)\vec b_2 + (u_1 p_2 cq_1 - u_2 p_1)\vec b_3$$

$=$

Acceleration can be solved similar as $\vec v$ (except you must use the corresponding $\vec\alpha$ formulas)