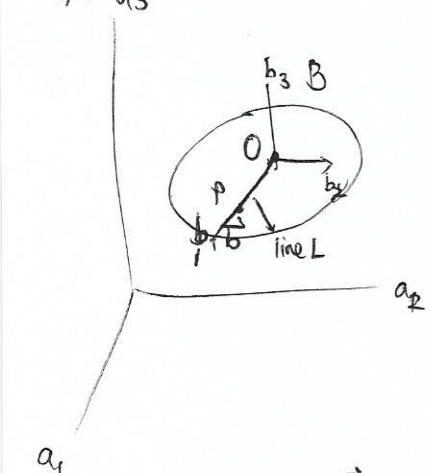
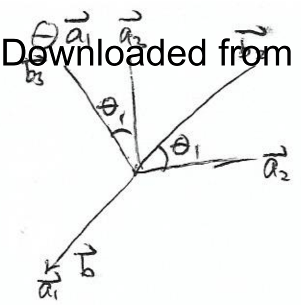
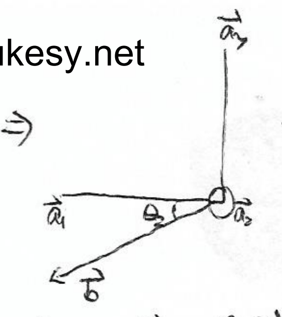
4.1



$\vec{p} = q\vec{b}$

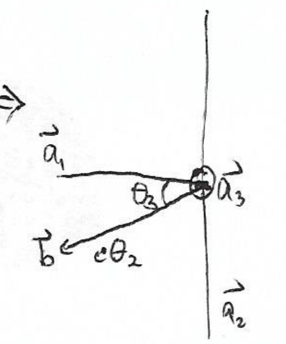


$\Rightarrow$



$\vec{b} = c\theta_2 \vec{a}_1 - s\theta_2 \vec{a}_3$

$\Rightarrow$



$\vec{b} = c\theta_2 c\theta_3 \vec{a}_1 + c\theta_2 s\theta_3 \vec{a}_2 - s\theta_2 \vec{a}_3$

If $\vec{p} = q\vec{b}$ for some constant q, then $\vec{p}$ will always be in line L

$$\vec{b} = \underbrace{c\theta_2 c\theta_3}_{x_1} \vec{a}_1 + \underbrace{c\theta_2 s\theta_3}_{x_2} \vec{a}_2 \underbrace{- s\theta_2}_{x_3} \vec{a}_3$$

relationship between x_1 and x_2

$$\frac{x_1}{x_2} = \frac{\cancel{c\theta_2} c\theta_3}{\cancel{c\theta_2} s\theta_3} \Rightarrow s\theta_3 x_1 - c\theta_3 x_2 = 0$$

relationship between x_1 and x_3

$$\frac{x_1}{x_3} = \frac{c\theta_2 c\theta_3}{-s\theta_2} \Rightarrow -s\theta_2 x_1 - c\theta_2 c\theta_3 x_3 = 0$$

If $\vec{p} = q\vec{b}$

$$\boxed{x_1 = q c\theta_2 c\theta_3 \quad x_2 = q c\theta_2 s\theta_3 \quad x_3 = -q s\theta_2}$$

4.2

$f_j(x_1, y_1, z_1, x_2, y_2, z_2)$ Find 4 holonomic constraint equations:

↳ 2 length of p_1 and p_2

↳ perpendicular to a_2

$$\boxed{\begin{aligned} f_1 &= x_1^2 + y_1^2 - L_1^2 \\ f_2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 - L_2^2 \\ f_3 &= z_1 \\ f_4 &= z_2 \end{aligned}}$$

4.3 $\boxed{5}$ constraint equations

2 from length of p_1 and p_2

2 from perpendicular to a_2

1 from distance between Q and P_2

4.4 let n be the # of generalized coordinates

$n = 3v - M = 3(1) - 2 = 1$ $\blacksquare$

$v = 1$ because there is only 1 point in the set

$M = 2$ bec. there are 2 constraint equations as shown in Prob 4.1

$$\vec{P} = f(q,t) = q\,c\theta_2(t)\,c\theta_3(t)\,\vec{a_1} + q\,c\theta_2(t)\,s\theta_3(t)\,\vec{a_2} - q\,s\theta_2(t)\,\vec{a_3} \quad \blacksquare$$

Notice that $\vec{P}$ is a function of (only) q and t.

4.5 a) show that the 4 constraint equations are satisfied

$$f_1 = x_1^2 + y_1^2 - L_1^2 = q_1^2 + (L_1^2 - q_1^2)^{1/2 \cdot 2} - L_1^2 = 0 \quad \blacksquare$$

$$f_2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 - L_2^2 = (q_2 - q_1)^2 + \left(L_2^2 - (q_2 - q_1)^2\right)^{1/2 \cdot 2} - L_2^2 = 0 \quad \blacksquare$$

$$f_3 = z_1 = 0 \quad \blacksquare$$

$$f_4 = z_2 = 0 \quad \blacksquare$$

b) Generalized coordinates must be able to express each of $x_i\ y_i\ z_i$ as a single valued func. in a given domain. But for each q_1, there are 2 possible y_1 (assuming real y_1). See example below,

$$3 = (5^2 - 4^2)^{1/2} \quad \text{and} \quad -3 = (5^2 - 4^2)^{1/2}$$

Similar w/ q_2. Hence q_1 and q_2 are not generalized coordinates

c) To show that θ_1 and θ_2 are generalized coor. for P_1 and P_2, I'll express P_1 and P_2 ~~solely~~ as function of θ_1 and θ_2.

$x_1 = L_1 c\theta_1$ $\qquad$ $x_2 = L_1 c\theta_1 + L_2 c(\theta_1 + \theta_2)$

$y_1 = L_1 s\theta_1$ $\qquad$ $y_2 = L_1 s\theta_1 + L_2 s(\theta_1 + \theta_2)$

$z_1 = 0$ $\qquad$ $z_2 = 0$

~~the~~ These also satisfy the constraint eq. f_1 to f_4

4.6 a) express x_i y_i z_i as function of q_1 〈too long, gave up〉

$x_1 = L_1 c q_1$ $\qquad$ $x_2 =$

$y_1 = L_1 s q_1$ $\qquad$ $y_2 =$

$z_1 = 0$ $\qquad$ $z_2 = 0$

Note: x_2 and y_2 is solved from the constraint eq.

$x_2^2 + (y_2 - L_4)^2 = L_3^2$

$(x_2 - L_1 c q_1)^2 + (y_2 - L_1 s q_1)^2 = L_2^2$

b) Show that the ff. satisfy the holonomic constraints.

Note: $f_3 = z_1$ and $f_4 = z_2$ are trivially satisfied for both i and ii. The other constraints are

$f_1 = x_1^2 + y_1^2 - L_1^2$

$f_2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 - L_2^2$

$f_5 = x_2^2 + (y_2 - L_4)^2 - L_3^2$

i. $x_1 = L_1 c_1$ $\quad y_2 = L_1 s_1$ $\quad z_1 = 0$

$x_2 = L_1 c_1 + L_2 c_2$ $\quad y_2 = L_1 s_1 + L_2 s_2$ $\quad z_2 = 0$

$f_1 = L_1^2 c_1^2 + L_1^2 s_1^2 - L_1^2 = 0$ ■

$f_2 = (L_2 c_2)^2 + (L_2 s_2)^2 - L_2^2 = 0$ ■

$f_5 = (L_1 c_1 + L_2 c_2)^2 + (L_1 s_1 + L_2 s_2 - L_4)^2 - L_3^2 = 0$

$\Rightarrow (L_1 c_1)^2 + 2 L_1 L_2 c_1 c_2 + (L_2 c_2)^2 + (L_1 s_1)^2 + (L_2 s_2)^2 + (L_4)^2 + 2 L_1 L_2 s_1 s_2 - 2 L_1 L_4 s_1 - 2 L_2 L_4 s_2 - L_3^2 = 0$

$\Rightarrow L_1^2 + L_2^2 - L_3^2 + L_4^2 + 2 L_1 L_2 (c_1 c_2 + s_1 s_2) - 2 L_4 (L_1 s_1 + L_2 s_2) = 0$ ■

Notice:

$L_1^2 + L_2^2 + 2 L_1 L_2 (c_1 c_2 + s_1 s_2)$ = distance from O to P_2 (calc from L_1 and L_2)

$(L_4 + L_3 s\theta)^2 + (L_3 c\theta)^2$ = distance from O to P_2 calculated from L_4 and L_3 $= L_4^2 + L_3^2 + 2 L_3 L_4 s\theta$

$= L_3^2 + L_4 (L_4 + 2 L_3 s\theta)$

$2 L_1 s_1 + 2 L_2 s_2 - L_4$

ii. same x_1 y_1 z_1 $\quad x_2 = L_3 c_3$ $\quad y_2 = L_3 s_3 + L_4$ $\quad z_2 = 0$

$f_1 = 0$ ■ ; $f_2 = (L_3 c_3 - L_1 c_1)^2 + (L_3 s_3 + L_4 - L_1 s_1)^2 - L_2^2$; $f_3 = L_3^2 c_3^2 + L_3^2 s_3^2 - L_3^2 = 0$ ■

$\Rightarrow L_3^2 c_3^2 - 2 L_1 L_3 c_1 c_3 + L_1^2 c_1^2 + L_3^2 s_3^2 + L_4^2 + L_1 s_1^2 + 2 L_3 L_4 s_3 - 2 L_1 L_3 s_1 s_3 - 2 L_1 L_4 s_1 - L_2^2$

$\Rightarrow L_1^2 - L_2^2 + L_3^2 + L_4^2 - 2 L_1 L_3 (c_1 c_3 + s_1 s_3) + 2 L_4 (L_3 s_3 - L_1 s_1)$

~~Notice: $L_1^2 + L_3^2 - 2 L_1 L_3 (c_1 c_3 + s_1 s_3)$~~

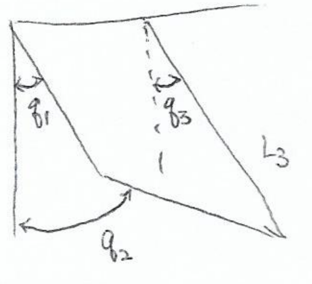
$f_2 = (\underbrace{L_2 c_2 + L_1 c_1 - L_1 c_1}_{\text{sub } ①})^2 + (\underbrace{L_1 s_1 + L_2 s_2 - L_1 s_1}_{\text{sub } ②})^2 - L_2^2 = 0$ ■

~~Holonomic~~

① $L_1 c_1 + L_2 c_2 - L_3 c_3 = 0$

② $L_1 s_1 + L_2 s_2 - L_3 s_3 - L_4 = 0$

c)



▸ Prob 4.3 only have 1 generalized coordinate hence only 1 of q_1 q_2 q_3 can be it.

▸ q_1 q_2 q_3 can express x_i y_i z_i hence all of them can be generalized coordinates.

4.7 (a) 6 for rigid body 1, 5 for body 2 9

(b) 6 for body 1, 1 for body 2 = 7 (c) 3 for position 3 for axis of rot = 6

(d) 2 (e) 1

4.8

from Prob 2.7 sol'n
$$\begin{cases} u_1 = {}^A\vec{\omega}^C \cdot \vec{b}_1 = -\dot{q}_2 \\ u_2 = {}^A\vec{\omega}^C \cdot \vec{b}_2 = \dot{q}_1 c q_2 \\ u_3 = {}^A\vec{\omega}^C \cdot \vec{b}_3 = \dot{q}_3 + \dot{q}_1 s q_2 \end{cases}$$

$$u_4 = {}^A\vec{V}^P \cdot \vec{a}_x = \dot{q}_4 + Rcq_1(u_3 - u_2 \tan q_2) = \dot{q}_4 + Rcq_1 \dot{q}_3$$

$$u_5 = {}^A\vec{V}^P \cdot \vec{a}_y = \dot{q}_5 + Rsq_1(u_3 - u_2 \tan q_2) = \dot{q}_5 + Rsq_1 \dot{q}_3$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ cq_2 & 0 & 0 & 0 & 0 \\ sq_2 & 0 & 1 & 0 & 0 \\ 0 & 0 & Rcq_1 & 1 & 0 \\ 0 & 0 & Rsq_1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{matrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{matrix} = \begin{bmatrix} 0 & \sec q2 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & \tan q2 & 1 & 0 & 0 \\ 0 & R\cos q_1 \tan q_2 & -R\cos q1 \ \cancel{\tan q2} & 1 & 0 \\ 0 & Rsq_1 \tan q_2 & -Rsq_1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}$$

If $q_2 = \pi/2$ rad, $\tan q_2$ goes to infinity so as long as $q_2 \neq \pi/2$ rad, $u_1 \cdots u_5$ are generalized ~~coordinates~~ speeds for C in A.

4.9 (a) $u_i = {}^N\vec{\omega}^B \cdot \vec{b}_i$

$$\left.\begin{aligned} u_1 &= \dot{q}_1 cq_2 + \dot{q}_3 - \Omega cq_1 sq_2 \\ u_2 &= \dot{q}_1 sq_2 sq_3 + \dot{q}_2 cq_3 + \Omega(sq_1 cq_3 + cq_1 cq_2 sq_3) \\ u_3 &= \dot{q}_1 sq_2 cq_3 - \dot{q}_2 sq_3 + \Omega(-sq_1 sq_3 + cq_1 cq_2 cq_3) \end{aligned}\right\} \text{from 2.9 sol'n}$$

$$\boxed{\begin{aligned} Z_1 &= -\Omega\, cq_1 sq_2 \\ Z_2 &= \Omega(sq_1 cq_3 + cq_1 cq_2 sq_3) \\ Z_3 &= \Omega(-sq_1 sq_3 + cq_1 cq_2 cq_3) \end{aligned}}$$

(b) $u_i = {}^A\vec{\omega}^B \cdot \vec{b}_i$

${}^A\vec{\omega}^B = \dot{q}_1 \vec{a}_1 + \dot{q}_2 \vec{b}_2^{(2)} + \dot{q}_3 \vec{b}_1$ } from 2.9 sol'n

Notice that even if I transform $\vec{a}_1$ and $\vec{b}_2^{(2)}$ to $\vec{b}_1, \vec{b}_2, \vec{b}_3$, it will have $\dot{q}_1$ and $\dot{q}_2$ components making it go to Yrs. Since all u_i has $\dot{q}_i$ components $\boxed{Z_1 = Z_2 = Z_3 = 0}$

4.10 From sol'n 3.8 $u_4 = R cq_1 \cancel{\dot{q}_3}(u_2 \tan q_2 - u_3)$ From sol'n 4.8 $u_1 = -\dot{q}_2$ $\quad u_3 = \dot{q}_3 + \dot{q}_1 sq_2$

$u_5 = R sq_1 \cancel{\dot{q}_3}(u_2 \tan q_2 - u_3)$ $\quad u_2 = \dot{q}_1 cq_2$

Eq. 2.13.1 $u_r = \sum_{s=1}^{p} A_{rs} u_s + B_r \Rightarrow$

$$\boxed{\begin{bmatrix} u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} 0 & R cq_1 \tan q_2 & -R cq_1 \\ 0 & R sq_1 \tan q_2 & -R sq_1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$$ //

4.11 $u_4 = {}^A\vec{\omega}^{C_1} \cdot \vec{n}_3 \qquad u_5 = {}^A\vec{\omega}^{C_2} \cdot \vec{n}_3 \qquad u_6 = {}^A\vec{v}^{S^*} \cdot \vec{n}_3$

Solve for velocity of $\hat{C}_1$ and $\hat{C}_2$ (point at C_1 and C_2 touching P)

$${}^A\vec{v}^{\hat{C}_1} = {}^A\vec{v}^{C^*} + \underbrace{{}^A\vec{\omega}^{C_1}}_{u_4 \vec{n}_3} \times -R\vec{n}_2$$

$$= {}^A\vec{v}^{S^*} + {}^A\vec{\omega}^{S} \times (-L\vec{n}_3) + u_4 \vec{n}_3 \times -R\vec{n}_2$$

$$= u_1 \vec{n}_1 + u_6 \vec{n}_3 - L u_2 \vec{n}_1 + L w_{as1} \vec{n}_2 + R u_4 \vec{n}_1$$

Note: ${}^A\vec{\omega}^S \times (-L\vec{n}_3) = \begin{vmatrix} w_{as1} & u_2 & u_3 \\ 0 & 0 & -L \end{vmatrix} = -L u_2 \vec{n}_1 + L w_{as1} \vec{n}_2$

Note: ${}^A\vec{v}^{S^*} = u_1 \vec{n}_1 + u_6 \vec{n}_3$ * no $\vec{n}_2$ component as C_1 and C_2 are at fixed height.

since C_1 rolls, ${}^A\vec{v}^{\hat{C}_1} = 0$ giving us $\underbrace{u_6 = 0}_{\vec{n}_3}$, $\underbrace{w_{as1} = 0}_{\vec{n}_2}$, $\underbrace{u_1 - L u_2 + R u_4 = 0}_{\vec{n}_1}$

$${}^A\vec{v}^{\hat{C}_2} = {}^A\vec{v}^{S^*} + {}^A\vec{\omega}^{S} \times (L\vec{n}_3) + u_5 \vec{n}_3 \times -R\vec{n}_2$$

$$= u_1 \vec{n}_1 + u_6 \vec{n}_3 + L u_2 \vec{n}_1 - L w_{as2} \vec{n}_2 + R u_5 \vec{n}_1$$

since C_2 rolls, ${}^A\vec{v}^{\hat{C}_2} = 0$ giving us $\underbrace{u_1 + L u_2 + R u_5 \vec{n}_1 = 0}_{\text{from } \vec{n}_1}$

Note:
${}^A\vec{\omega}^{C_1} \neq u_4 \vec{n}_3$
instead $= w_{c12} \vec{n}_2 + u_4 \vec{n}_3$
However ${}^A\vec{\omega}^{C_1} \times -R\vec{n}_2 = R u_4 \vec{n}_1$
so my sol'n is still consistent

$$\boxed{\begin{bmatrix} u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} -1/R & L/R & 0 \\ -1/R & -L/R & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$ //

4.12 $\quad {}^A\vec{\omega}^{C_1} = \omega_{c12}\vec{n}_2 + u_4\vec{n}_3 = {}^A\vec{\omega}^S + {}^S\vec{\omega}^{C_1} \Rightarrow u_4 = u_3 + \Omega_1$ from $\vec{n}_3$ eq

$$= u_2\vec{n}_2 + u_3\vec{n}_3 + \Omega_1\vec{n}_3$$

$${}^A\vec{\omega}^{C_2} = \omega_{c22}\vec{n}_2 + u_5\vec{n}_3 = {}^A\vec{\omega}^S + {}^S\vec{\omega}^{C_2} \qquad \Rightarrow \quad u_5 = u_3 + \Omega_2 \text{ from } \vec{n}_3 \text{ eq.}$$

$$= u_2\vec{n}_2 + u_3\vec{n}_3 + \Omega_2\vec{n}_3$$

From sol'n 4.11 we have

① $u_1 - Lu_2 + Ru_4 = 0$ $\qquad \Rightarrow \qquad$ ③ $u_1 - Lu_2 + Ru_3 + R\Omega_1 = 0$

② $u_1 + Lu_2 + Ru_5 = 0$ $\qquad\qquad\qquad$ ④ $u_1 + Lu_2 + Ru_3 + R\Omega_2 = 0$

③+④ : $2u_1 + 2Ru_3 + R(\Omega_1 + \Omega_2) = 0 \Rightarrow u_3 = -\frac{u_1}{R} - \frac{1}{2}(\Omega_1 + \Omega_2)$

$-$③+④ : $2Lu_2 - R(\Omega_1 - \Omega_2) = 0 \Rightarrow u_2 = \frac{R}{2L}(\Omega_1 - \Omega_2)$

$$\boxed{\begin{bmatrix} u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} 0 \\ -1/R \\ -1/R \\ -1/R \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} \frac{R}{2L}(\Omega_1 - \Omega_2) \\ -1/2(\Omega_1 + \Omega_2) \\ \frac{1}{2}(\Omega_1 - \Omega_2) \\ \frac{1}{2}(\Omega_2 - \Omega_1) \\ 0 \end{bmatrix}}$$

4.13 $\quad {}^A\vec{\omega}^C = u_1\vec{b}_1 + u_2\vec{b}_2 + u_3\vec{b}_3$ $\qquad$ from Prob 2.7 sol'n

from sol'n 3.3 $\quad {}^A\vec{v}^{C^*} = (\dot{q}_4 cq_1 - Ru_2\tan q_2 + \dot{q}_5 sq_1)\vec{b}_1 + (-\dot{q}_4 sq_1 sq_2 + \dot{q}_5 cq_1 sq_2)\vec{b}_2 + (\dot{q}_4 sq_1 cq_2 - \dot{q}_5 cq_1 sq_2 + Ru_1)\vec{b}_3$

from sol'n 3.6 $\quad {}^A\vec{v}^{\hat{C}} = (\dot{q}_4 + Rcq_1\dot{q}_3)\vec{a}_x + (\dot{q}_5 + Rsq_1\dot{q}_3)\vec{a}_y$

Transform ${}^A v^{C^*}$ and ${}^A v^{\hat{C}}$ in terms of u_i

$${}^A\vec{v}^{C^*} = R\vec{b}_3 u_1 - R\tan q_2\vec{b}_1 u_2 + \dot{q}_4\vec{a}_x + \dot{q}_5\vec{a}_y$$

$${}^A\vec{v}^{\hat{C}} = (Rcq_1 a_x + Rsq_1 a_y)(u_3 - u_2\tan q_2) + \dot{q}_4\vec{a}_x + \dot{q}_5\vec{a}_y$$

$$= R\vec{b}_1 u_3 - R\tan q_2\vec{b}_1 u_2 + \vec{a}_x\dot{q}_4 + \vec{a}_y\dot{q}_5$$

	${}^A\vec{\omega}_r^C$	${}^A v_r^{C^*}$	${}^A v_r^{\hat{C}}$
1	$\vec{b}_1$	$R\vec{b}_3$	0
2	$\vec{b}_2$	$-R\tan q_2\vec{b}_1$	$-R\tan q_2\vec{b}_1$
3	$\vec{b}_3$	0	$R\vec{b}_1$
4	0	$\vec{a}_x$	$\vec{a}_x$
5	0	$\vec{a}_y$	$\vec{a}_y$

Personal Note:
u_4 and u_5 seems to be diff. from 4.8 ...
$u_4 = \dot{q}_4 \quad u_5 = \dot{q}_5$
unless the $Rcq_1\dot{q}_3 / Rsq_1\dot{q}_3$ part somewhat cancels each other out...

4.14 From Sol'n 4.10 (3.8) $u_4 = Rcq_1(u_2 \tan q_2 - u_3)$

$u_5 = Rsq_1(u_2 \tan q_2 - u_3)$

Note: $u_4 \vec{a_x} + u_5 \vec{a_y} = (Rcq_1 \vec{a_x} + Rsq_1 \vec{a_y})(u_2 \tan q_2 - u_3) = R\vec{b_1} u_2 \tan q_2 - R\vec{b_1} u_3$

	${}^A\vec{\omega}_r^C$	${}^A V_r^{C^*}$	${}^A V_r^{\hat{C}}$
1	$\vec{b_1}$	$R\vec{b_3}$	0
2	$\vec{b_2}$	0	0
3	$\vec{b_3}$	$-R\vec{b_1}$	0

replacing $\vec{a_x} u_4 + \vec{a_y} u_5$ in terms of u_1 u_2 u_3 gives us the table to the left.

(Table 4.13 ans + $u_4 \vec{a_x} + u_5 \vec{a_y}$)

4.15 $\frac{{}^A d\vec{\beta}}{dt} \overset{\text{eq. } 1.9.1}{=} \sum_{r=1}^{n} \frac{{}^A \partial \vec{\beta}}{\partial q_r} \dot{q}_r + \frac{{}^A \partial \vec{\beta}}{\partial t} \overset{\text{eq. } 2.1.2}{=} \underbrace{{}^A\vec{\omega}^B}_{①} \times \vec{\beta}$

② ${}^A\vec{\omega}^B \overset{\text{eq. } 2.14.1}{=} \sum_{r=1}^{n} \vec{\omega}_r u_r + \vec{\omega}_t = \sum_{r=1}^{n} \vec{\omega}_r \dot{q}_r + \vec{\omega}_t$

$$\sum_{r=1}^{n} \frac{{}^A \partial \vec{\beta}}{\partial q_r} \dot{q}_r + \frac{{}^A \partial \vec{\beta}}{\partial t} = \overbrace{\left(\sum_{r=1}^{n} \vec{\omega}_r \dot{q}_r + \vec{\omega}_t\right)}^{\text{sub ② from ①}} \times \vec{\beta} \overset{\text{distributive}}{=} \sum_{r=1}^{n} (\vec{\omega}_r \times \vec{\beta}) \dot{q}_r + \vec{\omega}_t \times \vec{\beta}$$

Looking at $\dot{q}_i$'th component, we have

$$\frac{{}^A \partial \vec{\beta}}{\partial q_r} = \vec{\omega}_r \times \vec{\beta}$$

4.16 (a) $u_i \triangleq {}^N\vec{\omega}^B \cdot \vec{b_i} \Rightarrow {}^N\vec{\omega}_r^B = \vec{b_r}$ (trivially since ${}^N\vec{\omega}^B = u_1\vec{b_1} + u_2\vec{b_2} + u_3\vec{b_3}$)

(b) $u_i \triangleq {}^A\vec{\omega}^B \cdot \vec{b_i} \Rightarrow {}^N\vec{\omega}_r^B = \vec{b_r}$ but $\vec{\omega}_t$ won't be zero like in (a). Notice that ${}^A\vec{\omega}^B \cdot \vec{b_i}$ contains all components w/ $\dot{q}_r$, therefore ${}^N\vec{\omega}_r^B$ is similar to (a) (the only diff. w/ ${}^N\vec{\omega}^B$ is that ${}^N\vec{\omega}^B$ has none q_r components)

(c) $\hat{u}_i = \dot{q}_i \Rightarrow$

$$\boxed{\begin{aligned} {}^N\vec{\omega}_1^B &= cq_2\vec{b_1} + sq_2 sq_3 \vec{b_2} + sq_2 cq_3 \vec{b_3} \\ {}^N\vec{\omega}_2^B &= cq_3\vec{b_2} - sq_3\vec{b_3} \\ {}^N\vec{\omega}_3^B &= \vec{b_1} \end{aligned}}$$

based from Sol'n 2.9

(d) Expressing ${}^N\vec{\alpha}^B \cdot \vec{b_i}$ in terms of q_i u_i $\dot{u}_i$, definition (a) will lead to the simplest expression for α_i.

Why? (a) is simpler than (b) bec. for (a) ${}^N\vec{\omega}^B = u_1\vec{b_1} + u_2\vec{b_2} + u_3\vec{b_3}$ while (b), ${}^N\vec{\omega}^B = (u_1 + \_)\vec{b_1} + (u_2 + \_)\vec{b_2} + (u_3 + \_)\vec{b_3}$ where $\_$ is someth other variable. Differentiating the ${}^N\vec{\omega}^B$ of (a) is certainly much simpler.

(a) is simpler than (c) bec. ~~y~~ (a) can be expressed in terms of (c) (not simply)

(a) summarizes the information more compactly.

4.17

r	${}^E\omega_r^A$	${}^E\vec{\omega}_r^B = {}^E\omega_r^C = {}^E\omega_r^D$	${}^E\vec{v}_r^{A*}$	${}^E\vec{v}_r^{B*}$	${}^E\vec{v}_r^{C*}$	${}^E\vec{v}_r^{D*}$
1	$\vec{a}_1$	$c_1\vec{b}_1 + s_1\vec{b}_3$	$-L_A\vec{a}_2$	$Z_6\vec{b}_2$	$Z_0\vec{b}_2$	$Z_{13}\vec{b}_1 + Z_{15}\vec{b}_2 + Z_{16}\vec{b}_3$
2	0	$\vec{b}_2$	0	$L_8\vec{b}_1$	$Z_9\vec{b}_1$	$Z_{14}\vec{b}_1 - p_1\vec{b}_3$
3	0	0	0	0	$\vec{b}_3$	$\vec{b}_3$

Based from Sol'n 3.15.

4.18 $\quad {}^A\vec{V}^{C*} = -R\left[(u_1 sq_2 + u_3)\vec{b}_1 + u_2\vec{b}_3\right]$

Determine ${}^A\vec{V}_r^{C*} \cdot {}^A\vec{a}^{C*}$ by

(a) ${}^A\vec{V}_1^{C*} = -R sq_2 \vec{b}_1 \qquad {}^A\vec{V}_2^{C*} = -R\vec{b}_3 \qquad {}^A\vec{V}_3^{C*} = -R\vec{b}_1$

$$\frac{d\,{}^A\vec{V}^{C*}}{dt} = {}^A\vec{a}^{C*} = -R\left[\dot{u}_1 sq_2 \vec{b}_1 + u_1u_2 cq_2\vec{b}_1 + u_1 sq_2\dot{\vec{b}}_1 + \dot{u}_3\vec{b}_1 + u_3\dot{\vec{b}}_1 + \dot{u}_2\vec{b}_3 + u_2\dot{\vec{b}}_3\right]$$

$${}^A\vec{V}_1^{C*} \cdot {}^A\vec{a}^{C*} = R^2\left[\dot{u}_1 sq_2^2 + u_1u_2 sq_2 cq_2 + \dot{u}_3 sq_2 + u_1u_2 sq_2 cq_2\right]$$

Notice: $\vec{b}_1 \cdot \dot{\vec{b}}_1 = 0$, $\vec{b}_1 \cdot \dot{\vec{b}}_3 = \dot{q}_1 cq_2$ or $u_1 cq_2$

$${}^A\vec{V}_2^{C*} \cdot {}^A\vec{a}^{C*} = R^2\left[-u_1^2 sq_2 cq_2 - u_1u_3 cq_2 + \dot{u}_2\right]$$

Notice: $\dot{\vec{b}}_1 \cdot \vec{b}_3 = -\dot{q}_1 cq_2$ or $-u_1 cq_2$, $\dot{\vec{b}}_3 \cdot \vec{b}_3 = \cancel{-\dot{q}_2 sq_2 cq_2 + \dot{q}_2 sq_2 cq_2} = 0$

$${}^A\vec{V}_3^{C*} \cdot {}^A\vec{a}^{C*} = R^2\left[\dot{u}_1 sq_2 + 2u_1u_2 cq_2 + \dot{u}_3\right]$$

Note

$$\frac{d\vec{b}_1}{dt} = -\dot{q}_1 sq_1 \vec{a}_x + \dot{q}_1 cq_1 \vec{a}_y$$

$$\frac{d\vec{b}_3}{dt} = (\dot{q}_1 cq_1 cq_2 - \dot{q}_2 sq_1 sq_2)\vec{a}_x + (\dot{q}_1 sq_1 cq_2 + \dot{q}_2 cq_1 sq_2)\vec{a}_y + \dot{q}_2 cq_2 \vec{a}_z$$

$$\vec{b}_1 = cq_1\vec{a}_x + sq_1\vec{a}_y$$

$$\vec{b}_3 = sq_1 cq_2\vec{a}_x - cq_1 cq_2\vec{a}_y + sq_2\vec{a}_z$$

(b) $\quad {}^A\vec{V}^{C*2} = {}^A\vec{V}^{C*} \cdot {}^A\vec{V}^{C*} = R^2\left[(u_1 sq_2 + u_3)^2 + u_2^2\right] = R^2\left[u_1^2 sq_2^2 + 2u_1u_3 sq_2 + u_3^2 + u_2^2\right]$

$${}^A\vec{V}_r^{C*} \cdot {}^A\vec{a}^{C*} = \frac{1}{2}\left(\frac{d}{dt}\frac{\partial\vec{V}^2}{\partial\dot{q}_r} - \frac{\partial\vec{V}^2}{\partial q_r}\right) + \frac{1}{2}\sum_{s=p+1}^{n}\left(\frac{d}{dt}\frac{\partial\vec{V}^2}{\partial\dot{q}_s} - \frac{\partial\vec{V}^2}{\partial q_s}\right)A_{sr}$$

Note

$$\frac{\partial\vec{V}^2}{\partial u_1} = R^2\left[2u_1 sq_2^2 + 2u_3 sq_2\right]$$

$$\frac{d}{dt}\frac{\partial\vec{V}^2}{\partial u_1} = R^2\left[2\dot{u}_1 sq_2^2 + 4u_1 \overset{sq_2}{c q_2} u_2 + 2\dot{u}_3 sq_2 + 2u_2u_3 cq_2\right]$$

$$\frac{\partial\vec{V}^2}{\partial q_1} = 0$$

$$\frac{\partial\vec{V}^2}{\partial u_2} = R^2\, 2u_2$$

$$\frac{\partial\vec{V}^2}{\partial t\,\partial u_2} = R^2\, 2\dot{u}_2$$

$$\frac{\partial V^2}{\partial q_2} = R^2\left[2u_1^2 sq_2 cq_2 + 2u_1u_3 cq_2\right]$$

$$\frac{\partial\vec{V}^2}{\partial u_3} = R^2\left[2u_1 sq_2 + 2u_3\right]$$

$$\frac{\partial\vec{V}^2}{\partial t\,\partial u_3} = R^2\left[2\dot{u}_1 sq_2 + 2u_1u_2 cq_2 + 2\dot{u}_3\right]$$

$$\frac{\partial\vec{V}^2}{\partial q_3} = 0$$

Solving for $\frac{\partial\vec{V}^2}{\partial q_4}$ and $\frac{\partial\vec{V}^2}{\partial q_5}$ seems difficult (long process)